Turbulent dissipation of standing and propagating transverse waves in a structured medium

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6 December 2022

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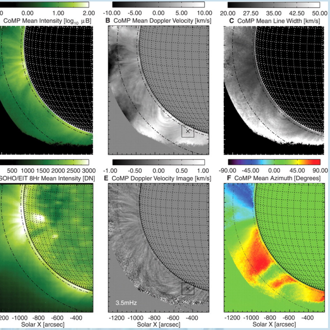
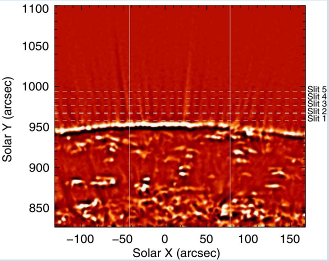
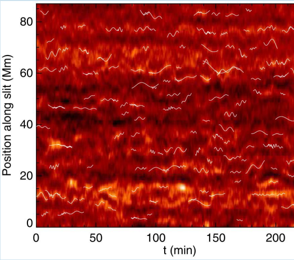
This project has received funding from the ERC (grant agreement No 724326)

Propagating transverse waves

Propagating transverse waves in the coronal loops and plumes
(Tomczyk et al. (2007), Tomczyk & McIntosh (2009), Thurgood et al. (2014))

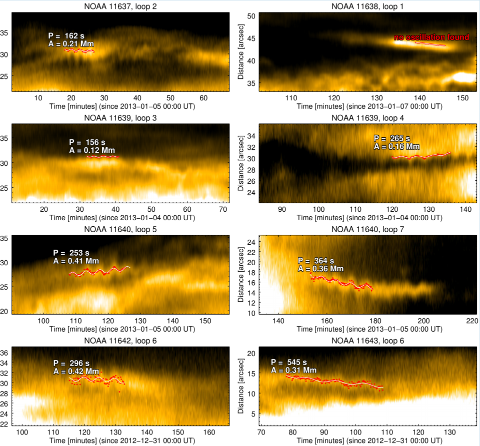


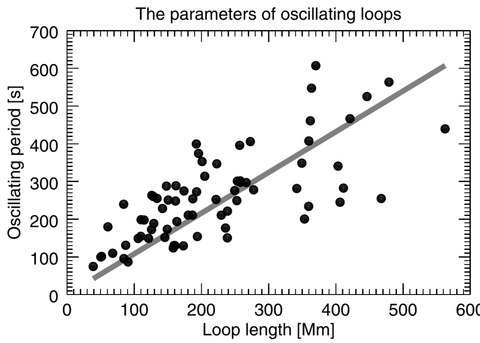




Standing transverse waves

Anfinogentov et al. (2015): decayless transverse waves in coronal loops are ubiquitous and standing (movie from Nisticò et al. 2013)





Alfvén turbulence

Upward and downward Alfvén waves described with Elsässer variables:

$$\vec{z}^{\pm} = \vec{v} \pm \frac{\vec{b}}{\sqrt{\mu\rho}}$$

Governing equations (incompressible MHD):

$$\frac{\partial \vec{z}^{\pm}}{\partial t} \mp \vec{v}_A \cdot \nabla \vec{z}^{\pm} = -\vec{z}^{\mp} \cdot \nabla \vec{z}^{\pm}$$

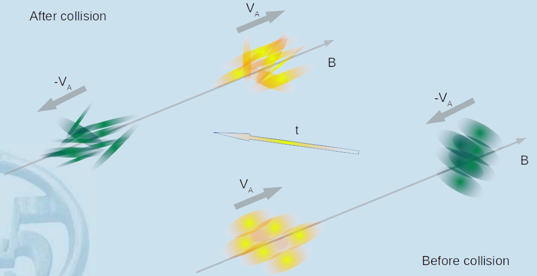
Classical thinking in solar wind models: Only non-linear evolution if both $\vec{z}^{+}$ and $\vec{z}^{-}$ are present.

Alfvén turbulence

Classical turbulence from counterpropagating Alfvén waves: ($\omega = \pm\omega_A$)

	Upward prop.	Downward prop.
z^-	x	
z^+		x

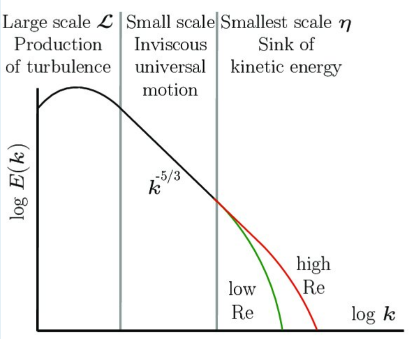
$\rightarrow$ non-linear term $\vec{z}^{\mp} \cdot \nabla \vec{z}^{\pm} \neq 0$



VD et al. (2020)

Alfvén turbulence

Alfvén turbulence leads to the formation of a power law:

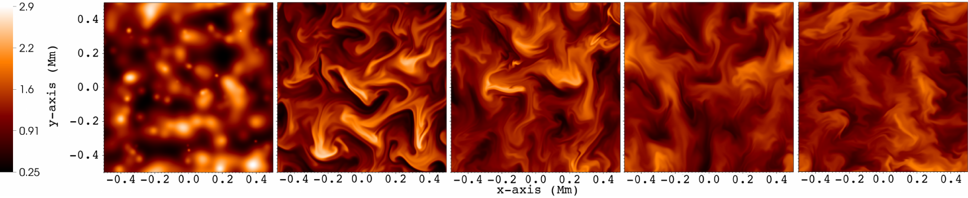


Stein (2013)

- Energy input at large scales by driver waves
- Energy cascade to small scales
  - with energy cascade rate ϵ independent of scale k
  - independent of dissipation mechanism and value
- Dissipation at small scales by dissipative effects

Uniturbulence

Magyar et al. (2017): $B = 5\mathrm{G}, \rho = 2\ 10^{-13}\mathrm{kg/m^3}$, 250 Gaussian density enhancements ("plumes"), drive with varying polarisation with RMS velocity of 12km/s.



Propagating waves (in one direction) form turbulent medium: uniturbulence (= turbulence from unidirectional waves)

Uniturbulence

Magyar et al. (2019b):

Linearised induction equation:

$$\frac{\partial \vec{B}'}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} \right)$$

Take $\vec{B} = B(x,y)\vec{e}_z$, Fourier analyse in z and t:

$$\frac{\partial \vec{B}'_\perp}{\partial t} = B(x,y)\frac{\partial \vec{v}_\perp}{\partial z}, \qquad -i\omega \vec{B}'_\perp = ikB(x,y)\vec{v}_\perp$$

Rearrange using $\vec{v} = \frac{1}{2}(\vec{z}^+ + \vec{z}^-)$ and $\vec{B}' = \frac{\sqrt{\mu\rho}}{2}(\vec{z}^+ - \vec{z}^-)$:

$$(\omega + \omega_A)\vec{z}^+_\perp = (\omega - \omega_A)\vec{z}^-_\perp$$

Also

$$\frac{\partial \vec{z}^\pm_\perp}{\partial t} = -\frac{\omega}{k}\frac{\partial \vec{z}^\pm_\perp}{\partial z}$$

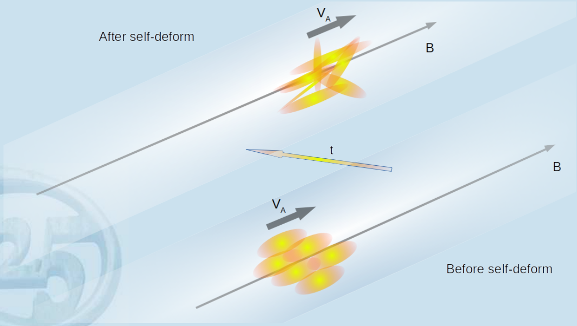
Unless $\omega = \pm\omega_A$, both z^+ and z^- in any wave! Self-cascade.

Alfvén turbulence

Uniturbulence from co-propagating transverse waves around inhomogeneity:($\omega \neq \pm\omega_A$)

	Upward prop.	Downward prop.
z^-	x	
z^+	x	

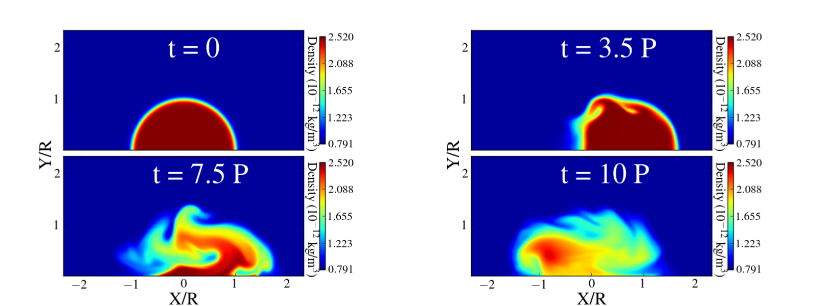
$\rightarrow$ non-linear term $\vec{z}^{\mp} \cdot \nabla \vec{z}^{\pm} \neq 0$



VD et al. (2020)

Driven transverse waves in loop

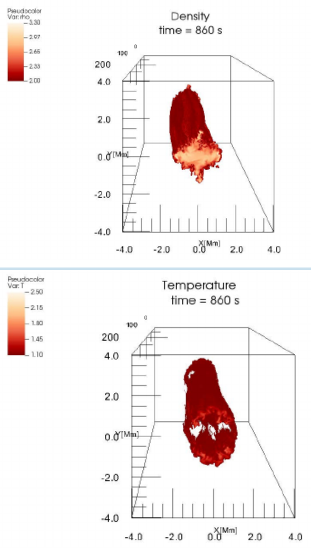
Karampelas et al. (2017), Karampelas & Van Doorsselaere (2018): loop with footpoint driver becomes fully turbulent (KHI)



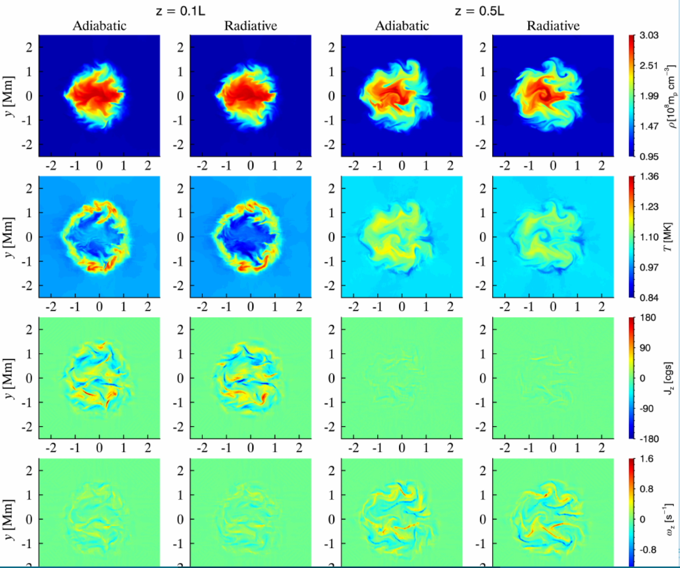
Loop heating against radiative losses

Shi et al. (2021):

- Straight density enhanced (contrast $= 3$, internal density 3e8 cm^{-3})
- temperature uniform loop (1MK)
- 200Mm, 30G
- Footpoint periodic velocity driver (8km/s, P=86s)
- Driving from $t = 0s$, radiative losses from $t = 600s$
- Background heating to keep exterior

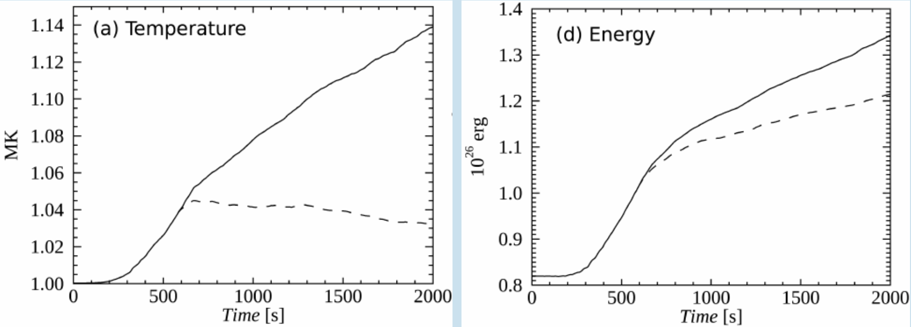


Loop heating against radiative losses



Loop heating against radiative losses

Shi et al. (2021): evolution of loop temperature and energy

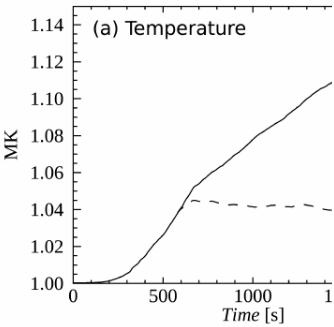


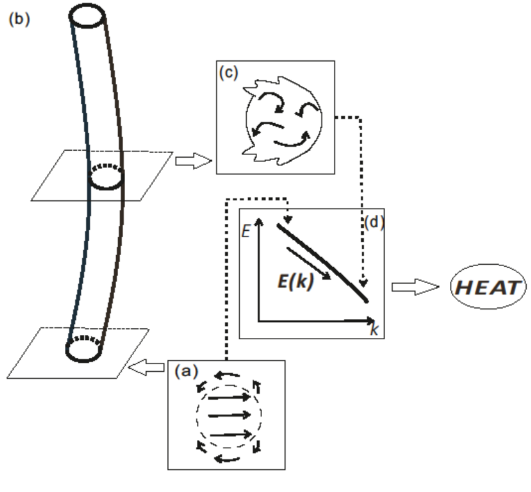
Wave heating

- supports (low density) loop against radiative losses,
- extends cooling time significantly.

Loop heating against

Shi et al. (2021): evolu





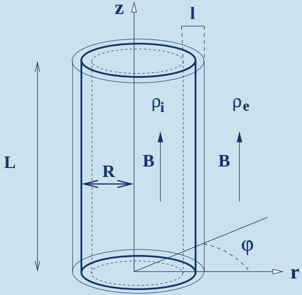
(a) Footpoint driver
(b) Fundamental standing kink wave
(c) KHI and Turbulence structure
(d) Energy cascade

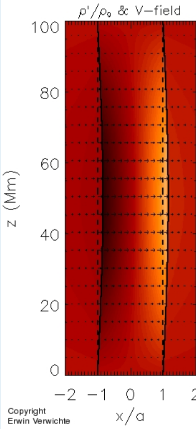
Wave heating

- supports (low dens
- extends cooling tin

Cylindrical coronal plume

- plasma cylinder of radius R, coordinates (r, ϕ, z)
- cold, ideal linearised MHD
- uniform $\vec{B} = B\vec{e}_z$
- high density inside ρ_i, low outside ρ_e





Movie by E. Verwichte

Kink waves in Elsässer terms

Start with Edwin & Roberts (1983) model of kink wave:
Standing waves

$$P'(r,\varphi,z,t) = \mathcal{R}(r)\cos\varphi\cos(k_z z)\cos(\omega t),$$

Propagating waves

$$P'(r,\varphi,z,t) = \mathcal{R}(r)\cos\varphi\cos(k_z z - \omega t),$$

with

$$\mathcal{R}(r) = \begin{cases} A\frac{J_1(\kappa_i r)}{J_1(\kappa_i R)} & \text{for } r \leq R, \\ A\frac{K_1(\kappa_e r)}{K_1(\kappa_e R)} & \text{for } r > R, \end{cases} \quad \text{and } \kappa_{\mathrm{i,e}}^2 = \left| \frac{\omega^2 - \omega_{\mathrm{A}}^2}{V_{\mathrm{A}}^2} \right|$$

and the usual dispersion relation:

$$\frac{\kappa_i}{\rho_i(\omega^2 - \omega_{\mathrm{Ai}}^2)} \frac{J_1'(\kappa_i R)}{J_1(\kappa_i R)} = \frac{\kappa_e}{\rho_e(\omega^2 - \omega_{\mathrm{Ae}}^2)} \frac{K_1'(\kappa_e R)}{K_1(\kappa_e R)},$$

Kink waves in Elsässer terms

The Elsässer variables:

$$z_r^{\pm} = \frac{\partial \mathcal{R}}{\partial r} \frac{1}{\rho_0(\omega^2 - \omega_{\mathrm{A}}^2)} \cos\varphi \begin{cases} -\omega \cos(k_z z)\sin(\omega t) \mp \omega_{\mathrm{A}} \sin(k_z z)\cos(\omega \\ (\omega \mp \omega_{\mathrm{A}})\sin(k_z z - \omega t) \end{cases}$$

$$z_\varphi^{\pm} = -\frac{\mathcal{R}}{r} \frac{1}{\rho_0(\omega^2 - \omega_{\mathrm{A}}^2)} \sin\varphi \begin{cases} -\omega \cos(k_z z)\sin(\omega t) \mp \omega_{\mathrm{A}} \sin(k_z z)\cos(\omega \\ (\omega \mp \omega_{\mathrm{A}})\sin(k_z z - \omega t) \end{cases}$$

$$z_z^{\pm} = \pm\left(\frac{\mu P'}{B_0}\frac{1}{\sqrt{\mu\rho_0}} - V_{\mathrm{A}}\frac{\rho'}{2\rho_0}\right) = \pm P'\left(\frac{1}{2\rho_0 V_{\mathrm{A}}}\right)$$

$$= \pm\frac{\mathcal{R}}{2\rho_0 V_{\mathrm{A}}} \cos\varphi \begin{cases} \cos(k_z z)\cos(\omega t) & \text{(standing)} \\ \cos(k_z z - \omega t) & \text{(propagating)} \end{cases}$$

BTW, satisfied Norbert's equation (for propagating waves):

$$(\omega + \omega_A)\vec{z}_\perp^{\,+} = (\omega - \omega_A)\vec{z}_\perp^{\,-}$$

Energy and energy dissipation rate

Following the solar wind turbulence theory, energy dissipation rate ϵ^- and energy density $w^\pm$ is:

$$\epsilon^- = \frac{\rho}{2}\vec{z}^-\cdot(\vec{z}^+\cdot\nabla\vec{z}^-) = \vec{z}^+\cdot\nabla\frac{\rho}{2}\frac{(z^-)^2}{2} = \vec{z}^+\cdot\nabla w^-, \text{ with } w^\pm = \frac{\rho(\vec{z}^\pm)^2}{4}$$

3rd order quantity! $\rightarrow$ averages to 0 over wave cycle
Use RMS average instead:

$$\langle\epsilon\rangle = \int_0^\infty r dr \left(\int_0^{2\pi} d\varphi \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt\ \epsilon^2\right)^{1/2}$$

Assumption: during half cycle with cascade, energy is immediately cascaded to higher order m
$\rightarrow$ damping needs to be fast enough

Energy and energy dissipation rate

Use thin tube approximation to progress ($\delta = R/L = k_z R/\pi \ll 1$):

$$\omega^2 = \omega_{\rm k}^2 = \frac{\rho_i \omega_{\rm A,i}^2 + \rho_e \omega_{\rm A,e}^2}{\rho_i + \rho_e}, \lim_{\delta \to 0} \mathcal{R}(r) = \mathcal{T}(r) = \begin{cases} A\frac{r}{R} & \text{for } r \leq R \\ A\frac{R}{r} & \text{for } r > R \end{cases}$$

For example, energy density (interior and exterior)

$$w_{\rm i}^{\pm} = \frac{1}{4}\frac{1}{\rho_{\rm i}(\omega^2 - \omega_{\rm Ai}^2)^2}\frac{A^2}{R^2}\begin{cases}(\omega\cos(k_z z)\sin(\omega t) \pm \omega_{\rm Ai}\sin(k_z z)\cos(\omega t))^2 \\ (\omega \mp \omega_{\rm Ai})^2 \sin^2(k_z z - \omega t)\end{cases}$$

$$w_{\rm e}^{\pm} = \frac{1}{4}\frac{1}{\rho_{\rm e}(\omega^2 - \omega_{\rm Ae}^2)^2}\frac{A^2 R^2}{r^4}\begin{cases}(\omega\cos(k_z z)\sin(\omega t) \pm \omega_{\rm Ae}\sin(k_z z)\cos(\omega \ldots \\ (\omega \mp \omega_{\rm Ae})^2 \sin^2(k_z z - \omega t)\end{cases}$$

Also expressions for kink wave pressure $P_{\rm k}$ and ϵ

Damping of propagating waves

Average over cross-section: radial variation does not lead to solar wind acceleration

e.g. Kink wave pressure P_{k}

$$\begin{aligned}\langle P_{\mathrm{k}} \rangle &= \iint P_{\mathrm{k}} r dr d\varphi \\ &= \iint P_{\mathrm{ki}} r dr d\varphi + \iint P_{\mathrm{ke}} r dr d\varphi \\ &= V^2 \pi R^2 \left(\frac{\rho_{\mathrm{i}} + \rho_{\mathrm{e}}}{2} \right) \begin{cases} \sin^2(k_z z) \cos^2(\omega t) \\ \sin^2(k_z z - \omega t) \end{cases},\end{aligned}$$

velocity amplitude V

Damping of propagating waves

Average of energy density and energy cascade rate:

For propagating waves, energy same as Goossens et al. (2013):

$$\langle w \rangle = \pi R^2 \frac{\rho_\mathrm{i} + \rho_\mathrm{e}}{2} V^2,$$

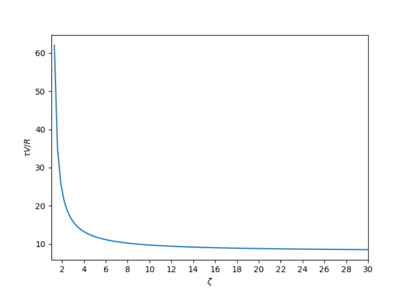
$$\langle \epsilon \rangle = V^3 \frac{\sqrt{5\pi} R}{10} \frac{\rho_\mathrm{e}}{\omega^3} |\omega(\omega^2 - \omega_\mathrm{Ae}^2)|,$$

Damping time:

$$\tau = \sqrt{5\pi} \frac{R}{V} \frac{2(\zeta + 1)}{|\zeta - 1|} = \sqrt{5\pi} \frac{P}{2\pi a} \frac{2(\zeta + 1)}{|\zeta - 1|}.$$

with density contrast $\zeta = \rho_\mathrm{i}/\rho_\mathrm{e}$, velocity amplitude V and maximal displacement $\eta = aR$.

Damping of propagating waves



Examples:

- Pant et al. (2019): velocity amplitude $V = 22\mathrm{km/s}$, radius $R = 250\mathrm{km}$
  $\rightarrow \tau \sim 180\mathrm{s}$ ($\zeta = 3$) or $\tau \sim 110\mathrm{s}$ ($\zeta = 10$)
- plumes with radius $R = 1\mathrm{Mm}$ and driver amplitude $V = 4\mathrm{km/s}$:
  $\rightarrow \tau \sim 3960\mathrm{s}$ ($\zeta = 3$)

Damping of standing waves

Energy density:

$$\langle w \rangle = \pi R^2 \frac{\rho_\mathrm{i} + \rho_\mathrm{e}}{4} V^2$$

Average energy dissipation rate:

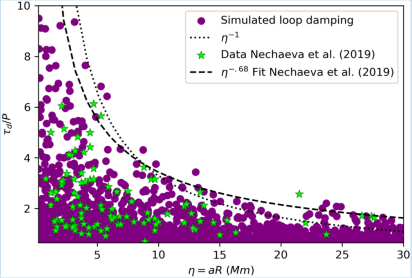
$$\langle\langle \epsilon \rangle\rangle = V^3 \frac{\sqrt{\pi} R}{10} \frac{\rho_\mathrm{e}}{8} \sqrt{\zeta^2 - 2\zeta + 97}$$

Damping time:

$$\tau = \frac{\langle w \rangle}{\langle\langle \epsilon \rangle\rangle} = 20\sqrt{\pi} \frac{R}{V} \frac{1+\zeta}{\sqrt{\zeta^2 - 2\zeta + 97}} = 20\sqrt{\pi} \frac{P}{2\pi a} \frac{1+\zeta}{\sqrt{\zeta^2 - 2\zeta + 97}}$$

Comparison with observations

VD et al. (2021): comparison with data of Nechaeva et al. (2019, green dots)



Purple dots: 5000 simulated loops with damping formula

- density contrast ζ drawn from $U[1, 9.5]$ (see results of Verwichte et al. 2013)
- thickness inhomogeneous layer l/R drawn from $U[0, 2]$,
- amplitude A drawn from $U[0.2, 30]\mathrm{Mm}$,
- radius R drawn from $U[0.5, 5]\mathrm{Mm}$.

Take minimum damping time of resonant absorption and non-linear damping.

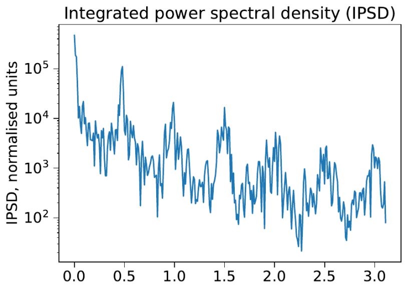
Add 50% noise to damping time. Method similar to Verwichte et al. (2013)

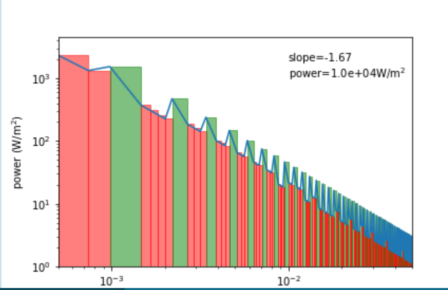
Heating function

- Use expressions of VD et al. (2020, 2021) for ϵ and take it as heating

$$\langle \epsilon \rangle = V^3 \frac{\sqrt{\pi} R}{10} \frac{\rho_\mathrm{e}}{\omega^3} \begin{cases} |\omega \cos(k_z z)| \sqrt{4\omega^4 \cos^4(k_z z) + (\omega^2 \cos^2(k_z z) - \omega_\mathrm{Ae}^2 \sin^2(k_z \ldots} \\ \sqrt{5} |\omega(\omega^2 - \omega_\mathrm{Ae}^2)| \end{cases}$$

- Use power spectrum input as a driver ($E \sim f^\nu$) with resonant frequencies (Afanasyev et al. 2020)





Heating function

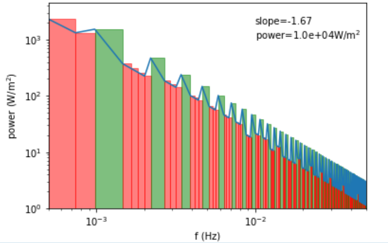
- input energy 10^4 W/m^2 $= \int_f E(f)df$
- heating $= \int_f E(f)\langle\epsilon\rangle(f,z)df$
- for each f:
  - if propagating wave (red):

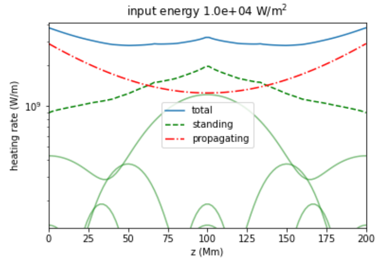
    $$\langle\epsilon\rangle(f,z) = \langle\epsilon\rangle_{\text{propagating}}(f)\exp\left(-z/V_{\text{ph}}\tau\right)$$

    where Poynting flux $= E(f)$
  - if standing wave (green):

    $$\langle\epsilon\rangle(f,z) = \langle\epsilon\rangle_{\text{standing}}(f,z)$$

    where amplitude such that $\int_z \langle\epsilon\rangle(f,z)dz = E(f)$





Towards UAWSOM

How can we use these calculations to extend the AWSOM model (or equivalent)? AWSOM uses:

- 2 fluid → perhaps MHD is enough?
- adds two extra equations for $w^{\pm}$: advection, reflection, cascade
- includes extra force due to Alfvén wave pressure P_{A}
- includes extra heating terms due to cascade

$$\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{P_i+P_e}{\gamma-1}+\frac{\rho u^2}{2}+\frac{B^2}{2\mu_0}+w_+ + w_-\right)\\
&+\nabla\cdot\left[\left(\frac{\rho u^2}{2}+\frac{\gamma(P_i+P_e)}{\gamma-1}+\frac{B^2}{\mu_0}\right)\mathbf{u}-\frac{\mathbf{B}(\mathbf{u}\cdot\mathbf{B})}{\mu_0}\right]\\
&+\nabla\cdot\left[\left(w_+ + w_- + P_A^{\mathrm{full}}\right)\mathbf{u}+(w_+ - w_-)\mathbf{V}_A\right]+Q_{\mathrm{noncons}}\\
&=-\nabla\cdot\mathbf{q}_e - Q_{\mathrm{rad}} - \rho\frac{GM_\odot}{r^3}\mathbf{r}\cdot\mathbf{u}, \qquad (29)
\end{aligned}$$

$$\begin{aligned}
Q_{\mathrm{noncons}} = \frac{\rho}{2}\Big[&\mathbf{z}_+\cdot(\mathbf{z}_-\cdot\nabla)\mathbf{u}+\mathbf{z}_-\cdot(\mathbf{z}_+\cdot\nabla)\mathbf{u}\\
&+\frac{\mathbf{z}_-\cdot(\mathbf{z}_+\cdot\nabla)\mathbf{B}-\mathbf{z}_+\cdot(\mathbf{z}_-\cdot\nabla)\mathbf{B}}{\sqrt{\mu_0\rho}}\Big]. \qquad (30)
\end{aligned}$$

$$\frac{\partial w_\pm}{\partial t}+\nabla\cdot[(\mathbf{u}\pm\mathbf{V}_A)w_\pm]+\frac{w_\pm}{2}(\nabla\cdot\mathbf{u}) = \mp\mathcal{R}\sqrt{w_-w_+}-\Gamma_\pm w_\pm, \qquad (36)$$

Towards UAWSOM

This work: new ingredients

- Kink wave pressure P_k (equiv. to Alfvén wave pressure P_A)
- Energy cascade rate kink waves ϵ (equiv. to Alfvén wave cascade $\Gamma^{\pm}w^{\pm}$)

How can we model kink wave reflection off an Alfvén speed gradient?

Is it equivalent to $\mathcal{R}\sqrt{w^+w^-}$ (van der Holst)?

Towards UAWSOM

Tentative idea:

$$\vec{z}^{\pm} = \vec{z}_A^{\pm} + \vec{z}_k^{\pm}, \text{ in } \frac{\partial \vec{z}^{\pm}}{\partial t} \mp \vec{v}_A \cdot \nabla \vec{z}^{\pm} = -\vec{z}^{\mp} \cdot \nabla \vec{z}^{\pm}$$

Resulting in terms:

- $z_A^{\mp} \cdot \nabla z_A^{\pm} \rightarrow$ Van der Holst
- $z_k^{\mp} \cdot \nabla z_k^{\pm} \rightarrow$ Van Doorsselaere
- $z_A^{\mp} \cdot \nabla z_k^{\pm}$ & $z_k^{\mp} \cdot \nabla z_A^{\pm}$: cascade of Alfvén/kink due to the other (probably $\neq 0$, Guo et al. 2019)

Do we also need reflected kink waves? $z_{\text{k,down}}^{\pm}$?

How do reflected kink waves interact with upward kink waves?

Conclusions

- Description of kink waves in Elsässer variables: standing and propagating
- Computation of solar wind like kink energy cascade rate
- Damping times V^{-1}
- Confirmed numerically (see talk Ismayilli)
- Compatible with observations (for standing kink wave damping)
- How can we use this for a UAWSOM solar wind model?

	MHD turbulence		Uniturbulence	
	Upward	Downward	Upward	Downward
z^-	✓		✓	
z^+		✓	✓	
	counterpropagating $\omega = \pm\omega_A$		co-propagating $\omega \neq \pm\omega_A$	