

# NI MHD Modeling of the Corona and Turbulence

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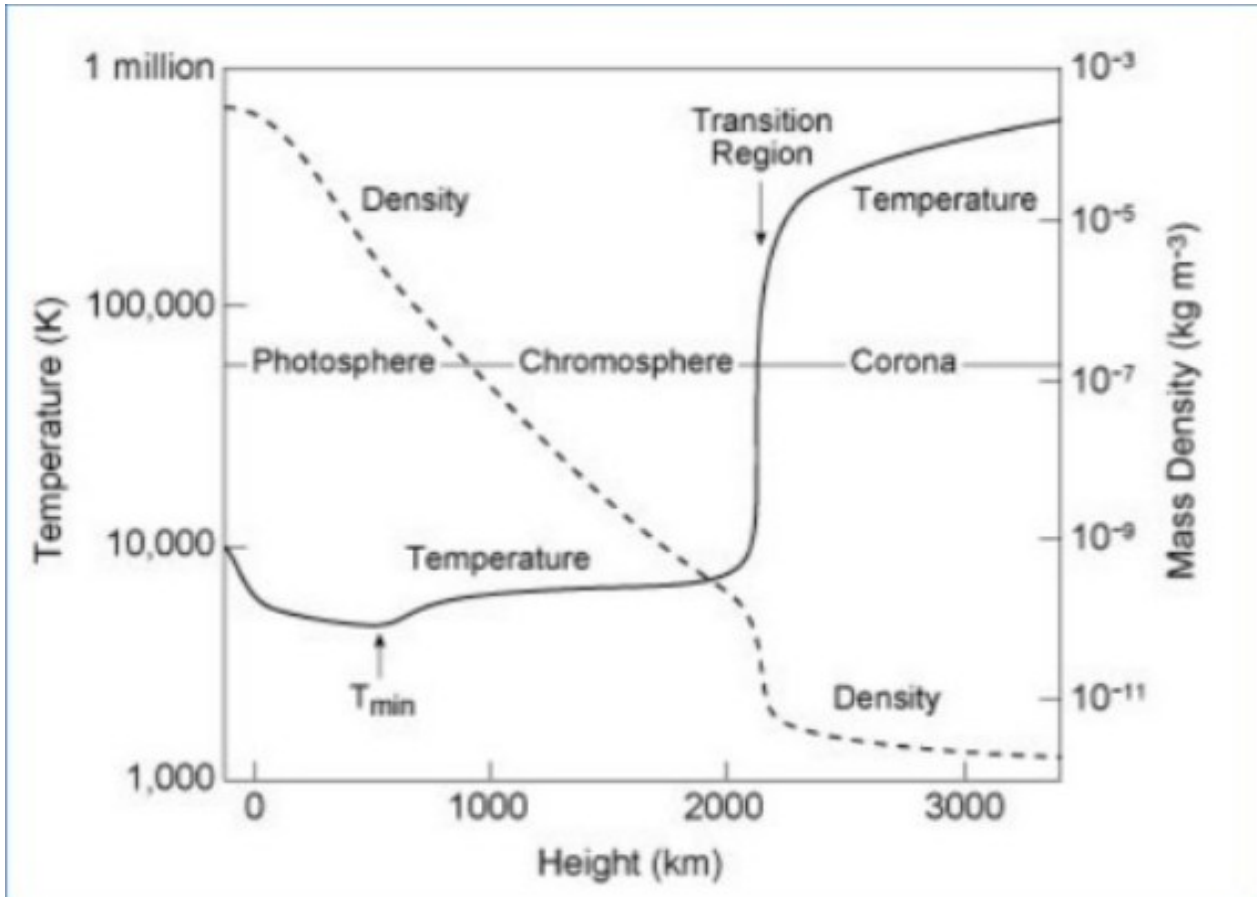


First meeting, ISSI, December 5 – 9, 2022

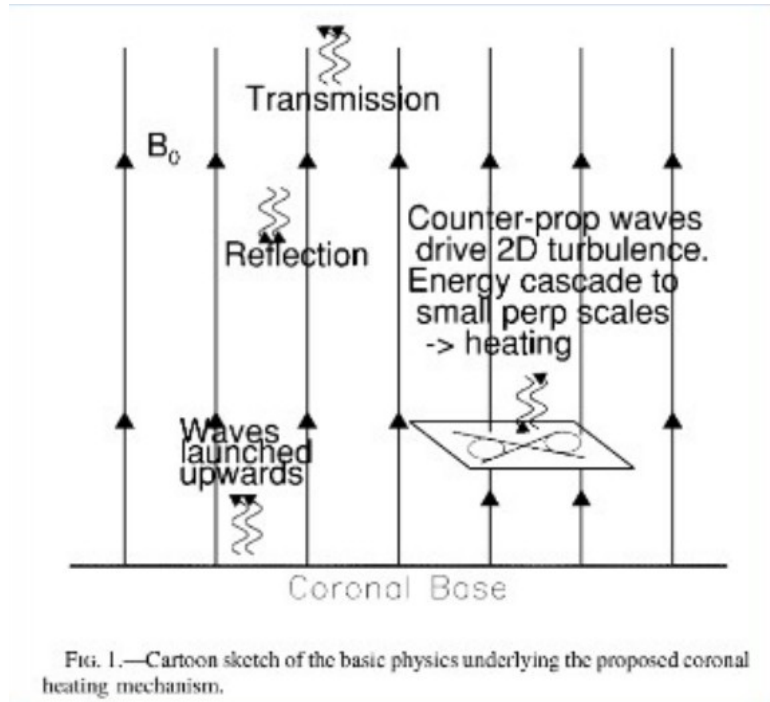


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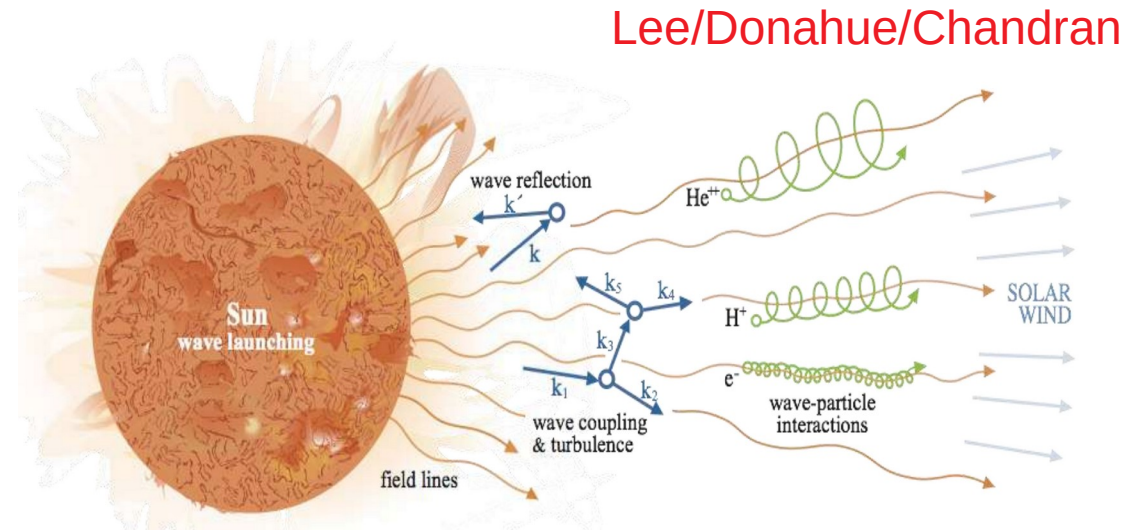
# Temperature of the solar corona



# Wave/Turbulence approach



(Matthaeus et al 1999)

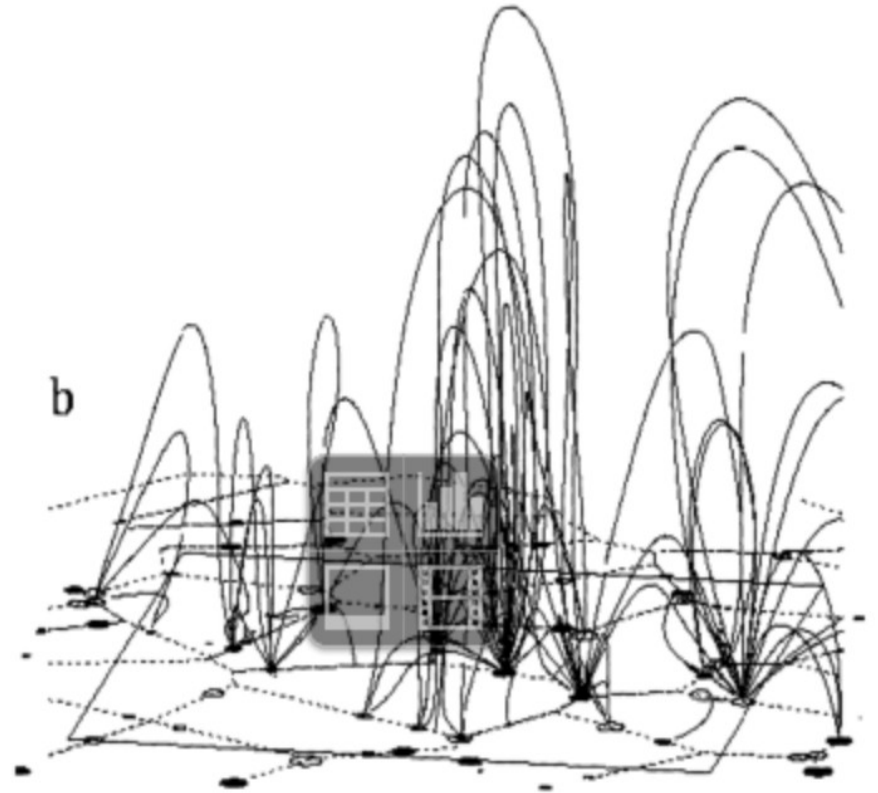


Many papers followed Matthaeus et al 1999 approach (e.g., Matsumoto & Shibata 2010; Verdini et al. 2010; Chandran et al. 2011; Lionello et al. 2014; Woolsey & Cranmer 2014)

# Reconnection/Loop opening approach

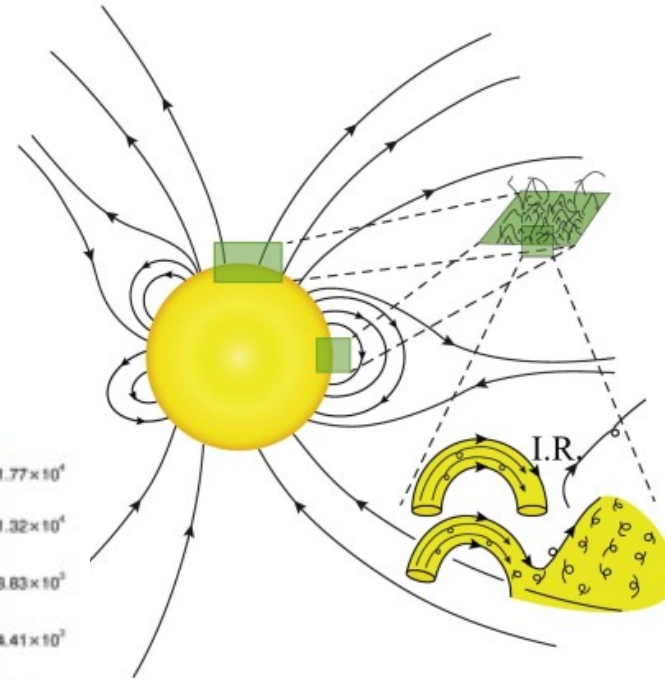
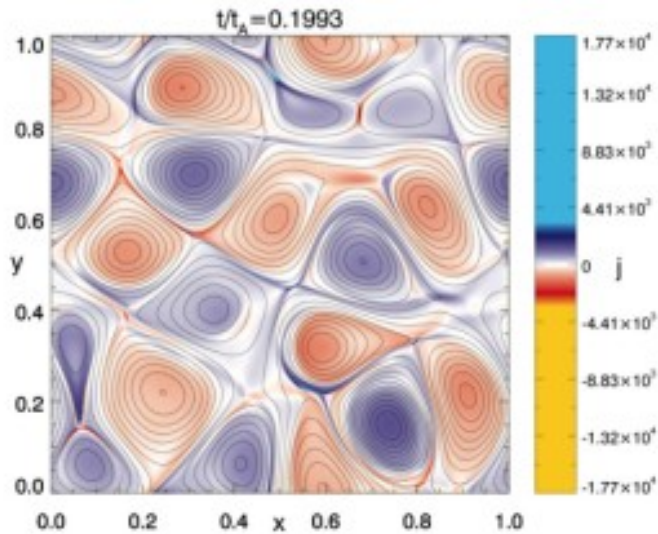
Transverse photospheric motions lead to “twisting and braiding” of magnetic field lines and energy released in form of current sheets, with dissipation thought to occur via nanoflares (Parker, 1972, 1988, 1994)

- Mass and energy deposited into open-field regions via interchange reconnection (Fisk et al., 1999, Fisk, 2003)
- Magnetic carpet (Title & Schrijver, 1998) - photosphere

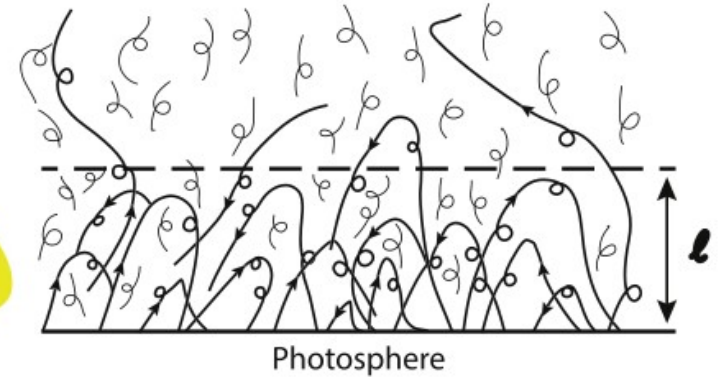


# Generation of 2D turbulence

(Rappazzo and Parker 2013)



Closely related to the ideas of Parker (1972, 1988, 1994); Fisk 2003, Fisk et al 1999).



(Zank et al 2018)

# 2D+slab vs W/T transport model of coronal turbulence

## NI 2D+slab model

- Majority 2D and minority slab in energy-containing range
- Anisotropic IR, primarily 2D, minority slab
- Uni-directional Alfvén/slab propagation with  $k_{\parallel}^{-5/3}$  spectra is possible
- $O(M)$  density fluctuations,  $k_p^{-5/3}$  spectra, and variance

## W/T model

- Majority slab and minority 2D in energy containing range
- Either isotropic IR likely or G-S scaling
- Uni-directional Alfvén/slab propagation with  $k_{\parallel}^{-5/3}$  spectra is not possible
- $O(M)$  density fluctuations

- 2D turbulence and NI/slab turbulence follow the ordering  $M_A^t : (M_A^t)^2$  (Zank et al 2017), where  $M_A^t$  is the turbulent Alfvén Mach number.



## Does Turbulence Turn off at the Alfvén Critical Surface?

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### NI 2D+slab model

- ➔  $\sigma_c^* = \pm 1$ ,  $\sigma_D^* = 0$ ,  $E_T^*$  arbitrary,
- ➔  $L^*$  arbitrary, and  $L_D^*$  arbitrary,
- ➔  $\sigma_c^\infty \neq \pm 1$ ,  $\sigma_D^\infty \neq 0$ ,  $E_T^\infty$  arbitrary,
- ➔  $L_\infty^\pm$  arbitrary, and  $L_D^\infty$  arbitrary.

➔  $NL_\pm^{\text{Total}} = -z^{\infty\pm} \frac{E_T^{\infty 1/2} |1 \mp \sigma_c^\infty|^{1/2}}{\lambda_\infty^\pm} - z^{*\pm} \frac{E_T^{\infty 1/2} |1 \mp \sigma_c^\infty|^{1/2}}{\lambda_\infty^\pm} \neq 0.$

Nonlinear  
dissipation term

- Turbulence does not turn off at the Alfvén critical surface/highly field-aligned flow

### W/T model

- ➔  $\sigma_c = -1$ ,  $\sigma_D$  arbitrary,  $E_T$  arbitrary,
- ➔  $L^+ = 0$ ,  $L^-$  arbitrary, and  $L_D$  arbitrary

Nonlinear dissipation term

➔  $NL_\pm = 0$  and  $NL_{\text{diss}}^{\text{Total}} = 0.$

- Turbulence turns off at the Alfvén critical surface/highly field-aligned flow

(See also Alberti et al 2022)

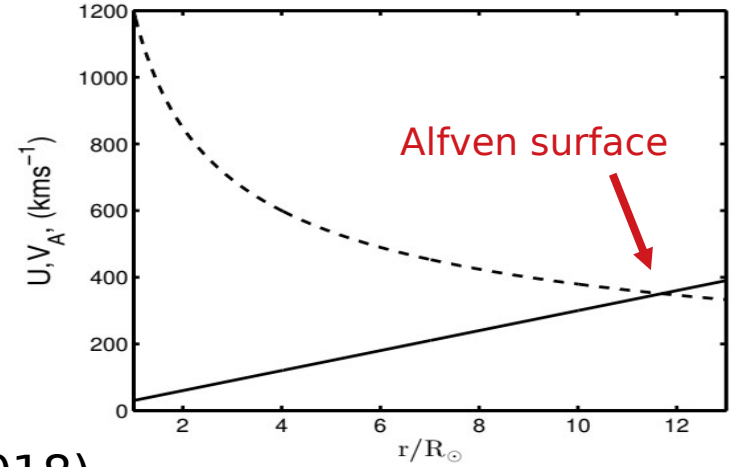
# NI MHD Coronal Turbulence Model

## Fixed background profile

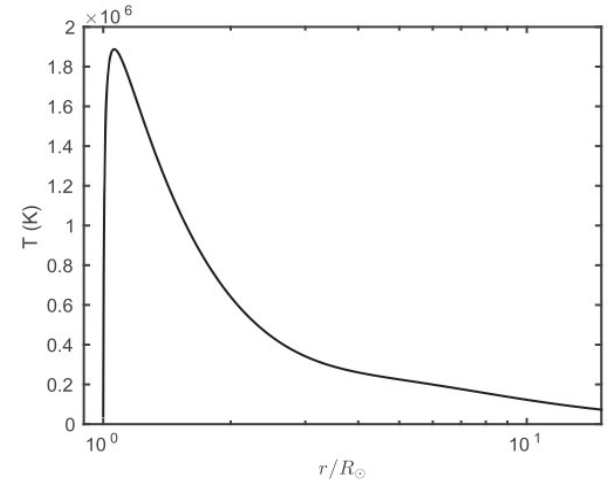
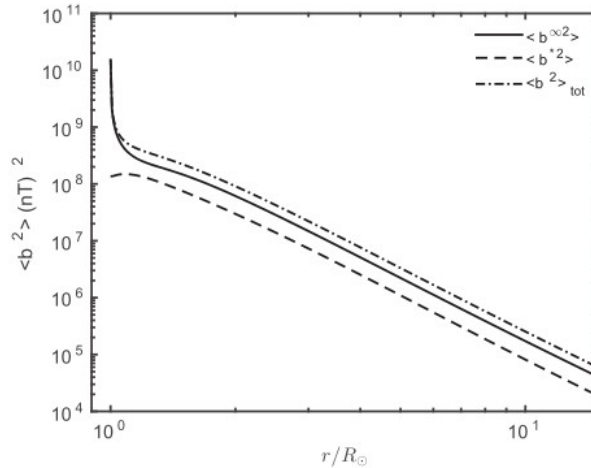
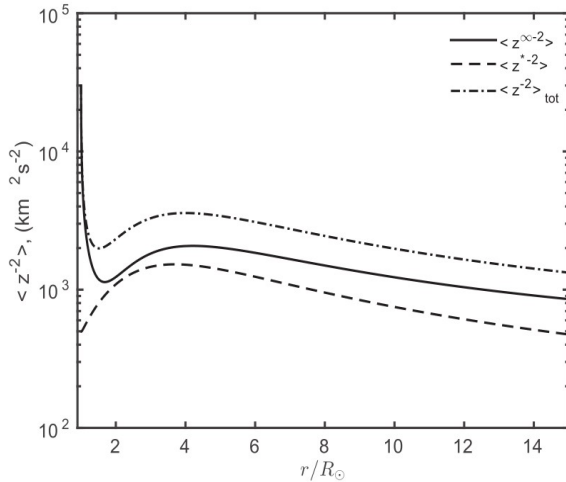
$$\mathbf{U} = U_0 \left( \frac{r}{r_0} \right) \hat{r} \quad \rho = \rho_0 \left( \frac{r_0}{r} \right)^3 \quad \mathbf{B} = B_0 \left( \frac{r_0}{r} \right)^2 \hat{r}$$
$$\mathbf{V}_A = \frac{\mathbf{B}}{\sqrt{\mu_0 \rho}} = \frac{B_0}{\sqrt{\mu_0 \rho_0}} \left( \frac{r_0}{r} \right)^{1/2} \hat{r} = V_{A0} \left( \frac{r_0}{r} \right)^{1/2} \hat{r}$$

where  $U_0 = 30$  km/s and  $V_{A0} = 900$  km/s

➔ Alfvén surface  $\sim 11.69 R_{\text{sun}}$

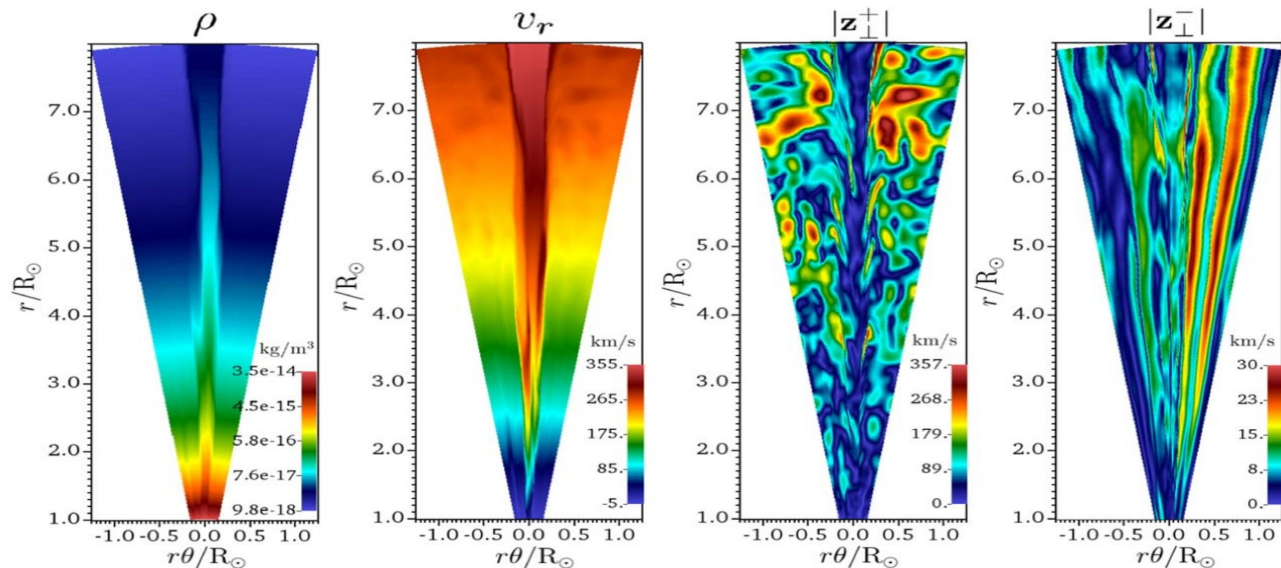


(Zank et al 2018)

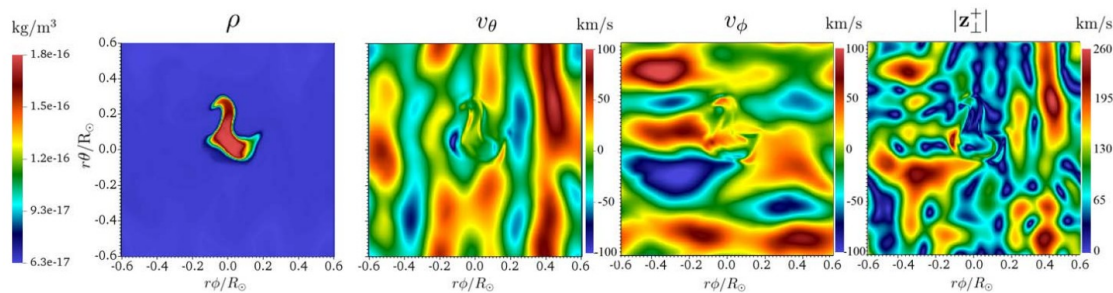




# Structures in the solar atmosphere

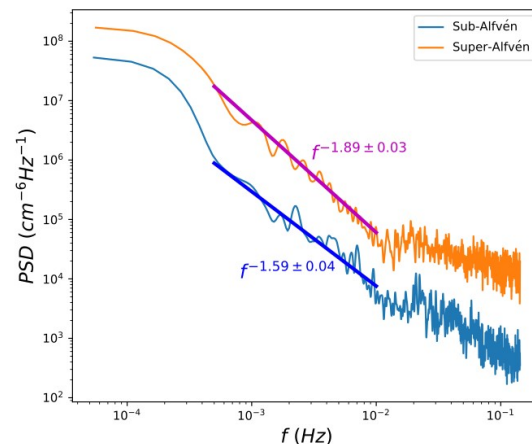


**Figure 14.** Snapshots of density ( $\rho$ ), radial velocity ( $v_r$ ), and the perpendicular Elsässer variables  $|z_{\perp}^+|$  and  $|z_{\perp}^-|$ , respectively (from left to right), in a slice at  $\phi = \pi/2$ ,  $1 t_A$  after the start of the simulation, in the stochastically driven plume simulation.



**Figure 15.** Snapshots of density ( $\rho$ ), perpendicular velocity components ( $v_{\theta}$  and  $v_{\phi}$ ), and the perpendicular Elsässer variable  $|z_{\perp}^+|$ , respectively (from left to right), in a spherical slice at  $r = 4 R_{\odot}$ ,  $1.5 t_A$  after the start of the simulation, in the stochastically driven plume simulation.

(Magyor et al 2021)

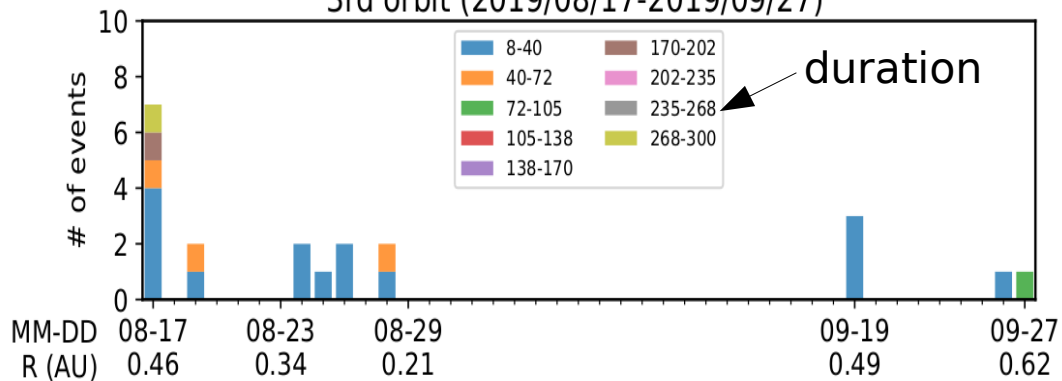


- Spacecraft cannot measure the 2D turbulence in the parallel geometry between the background fields. This does not mean that 2D turbulence is absent.
- The entropic density fluctuations are advected by the dominant 2D velocity fluctuations in the sub- and super-Alfvénic region during encounter 8 (where the background fields are parallel, and PSP observes only the slab component), consistent with the NI MHD description.

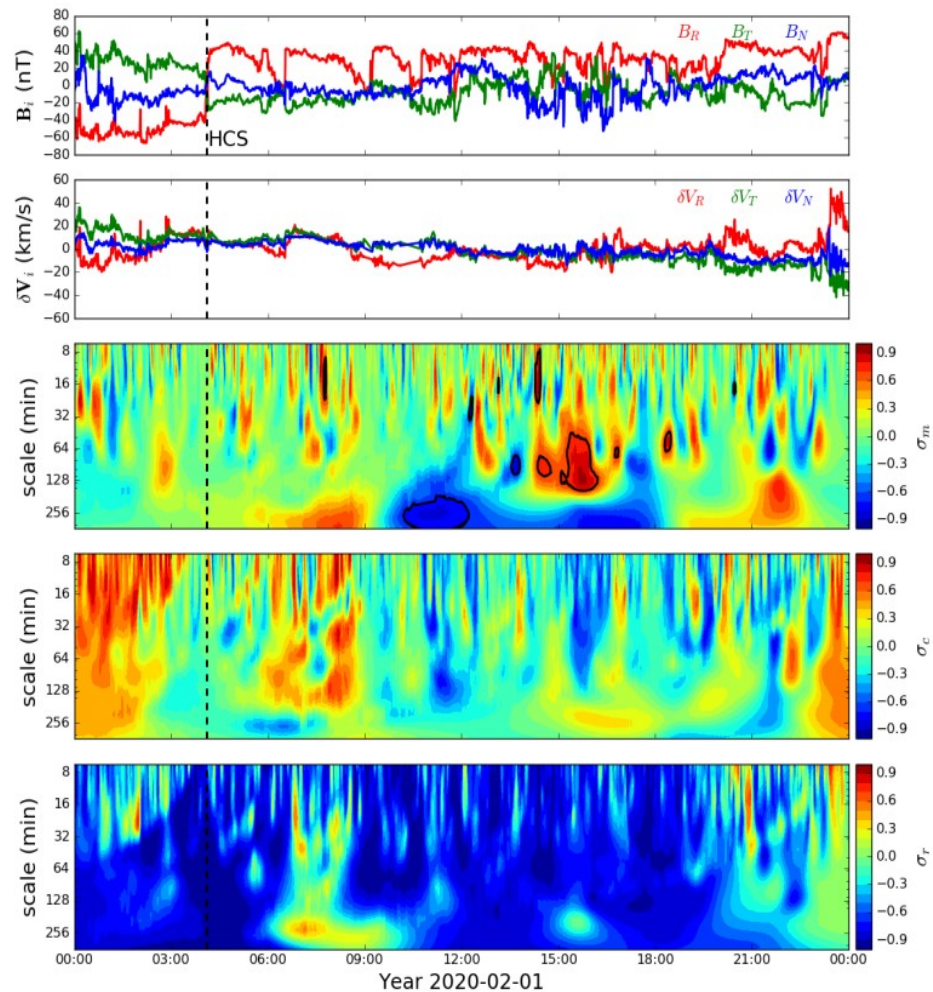
(Zank et al 2022)

# Structures near the Sun

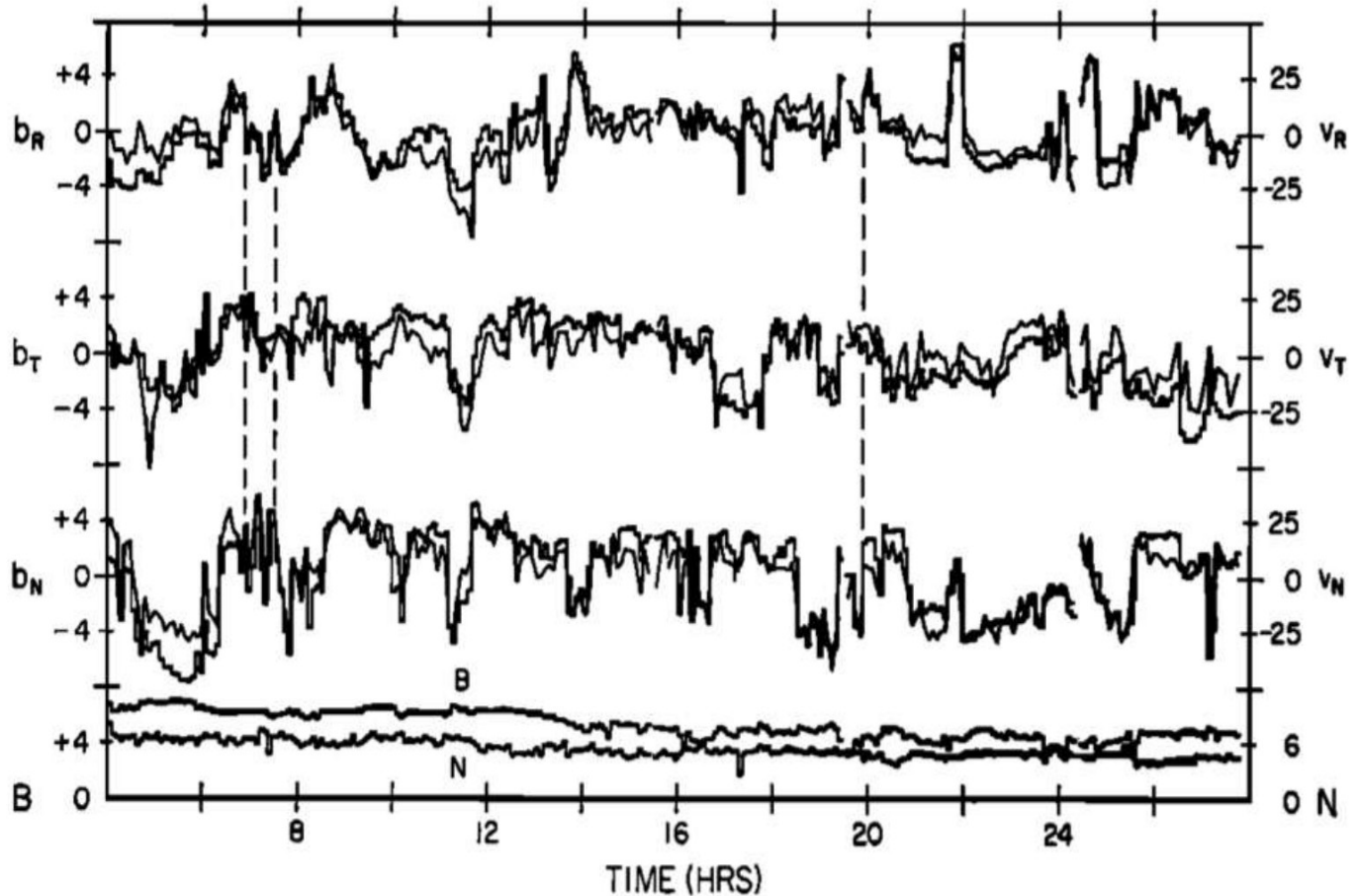
3rd orbit (2019/08/17-2019/09/27)



(Zhao et al 2021)



# Alfven waves in the solar wind



Velocity and magnetic field fluctuations are highly correlated.

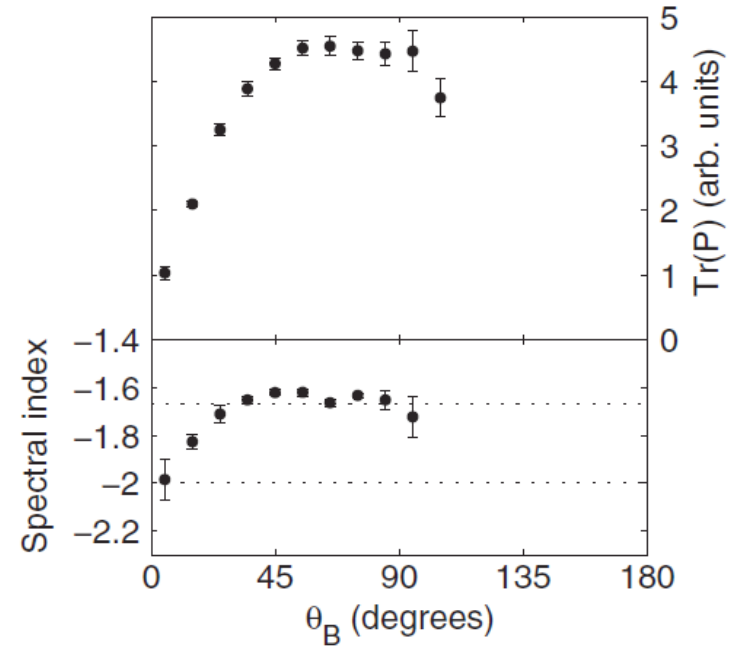
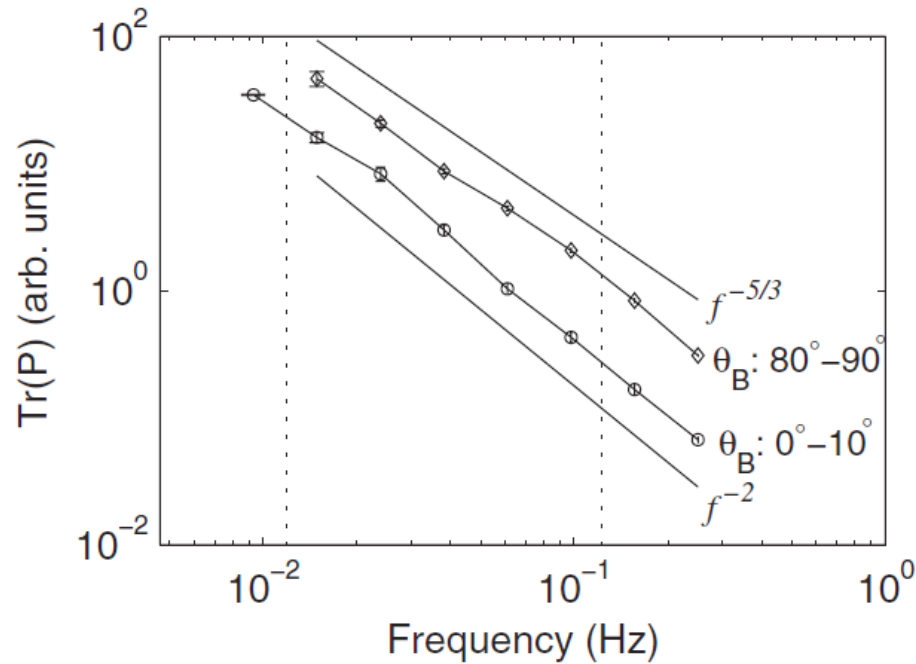


Pure Alfven wave

Does there exist a turbulence?

(Belcher & Davis 1971)

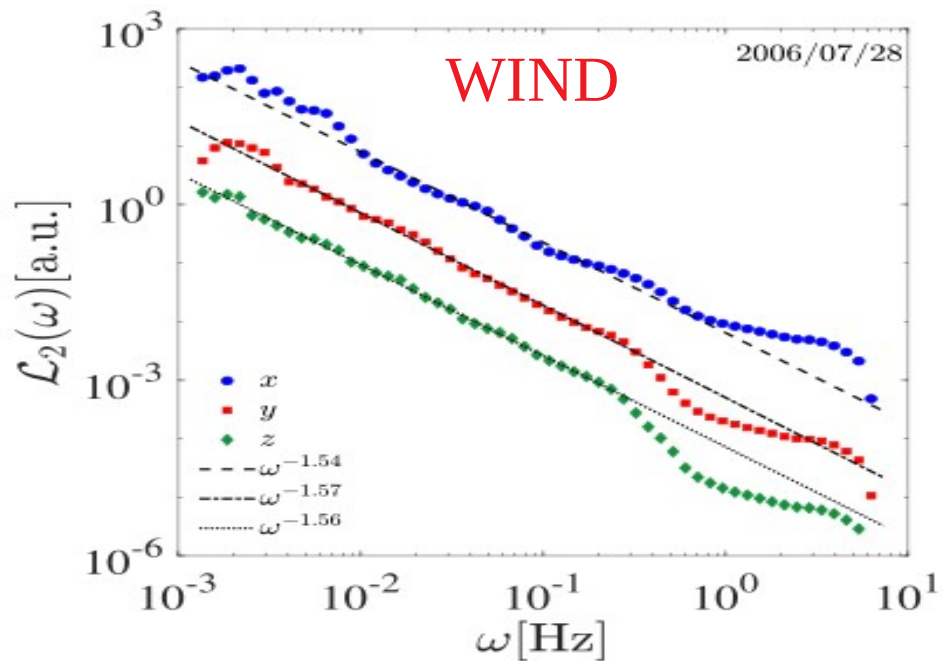
# Solar wind observation



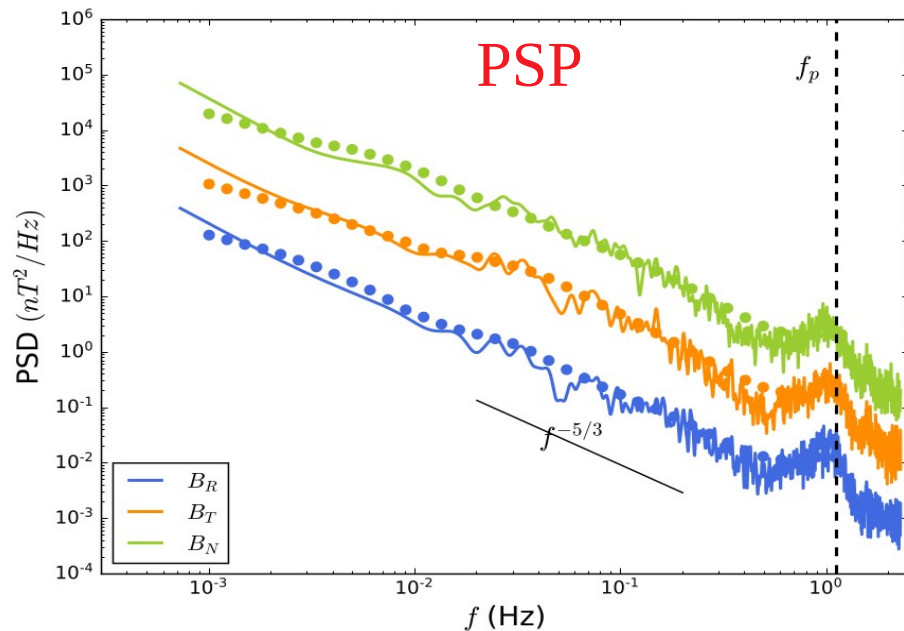
Horbury et al. (2008)

- One of the properties of solar wind turbulence is spectral anisotropy.
- Critical balance predicts different scalings for parallel and perpendicular spectra  $k_\perp^{-5/3}$  and  $k_\parallel^{-2}$  provided the (normalized) cross helicity  $|\sigma_c| \sim 0$ .
- Assuming Taylor's hypothesis and the Parker spiral magnetic field, PSP is more likely to measure parallel spectra  $k_\parallel$  close to the Sun.

# Highly field-aligned flow



(Telloni et al 2019)



(Zhao et al. 2020)

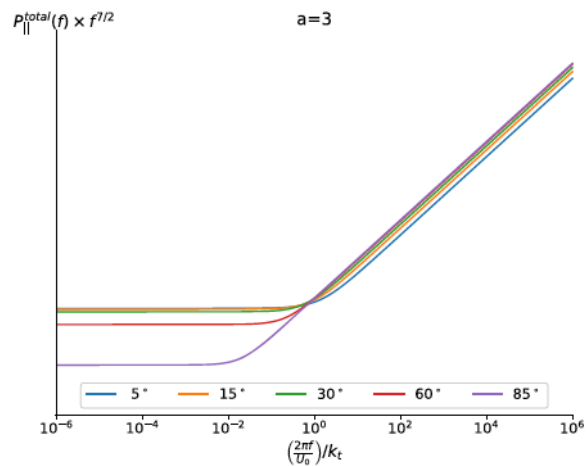
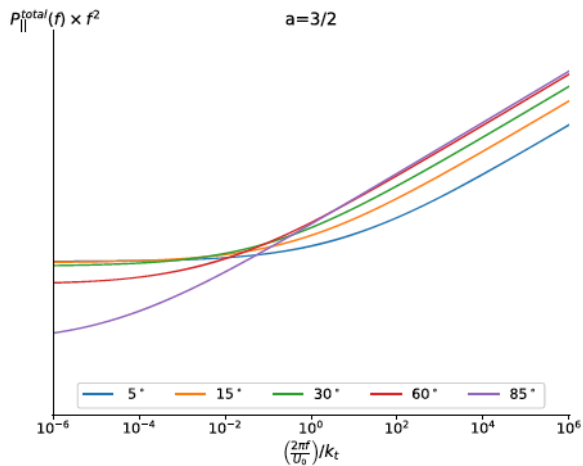
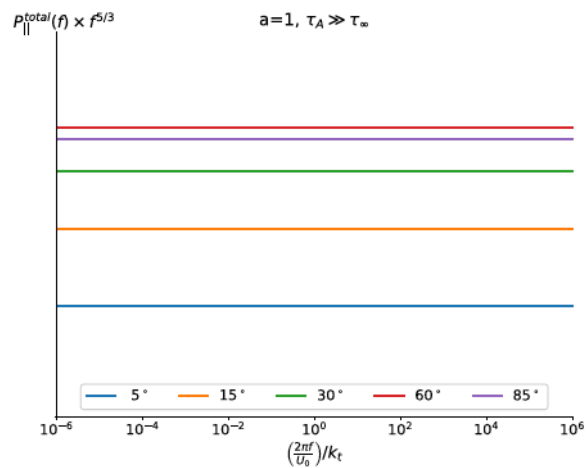
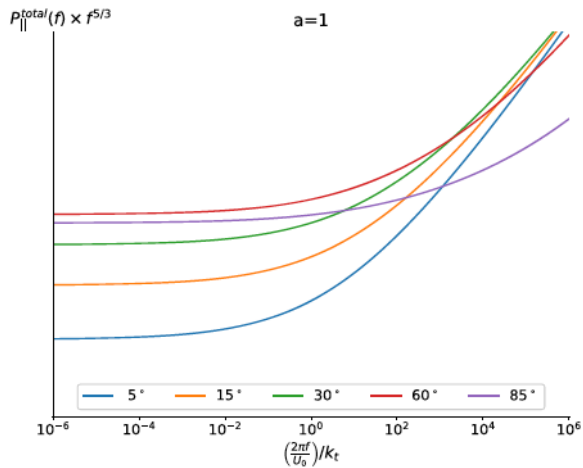
WIND & PSP observed highly field-aligned flow:

- the normalized cross helicity  $\sim 0.8 - 1$
- the spectra show a  $-5/3$  power law.

- Different from critical balance theory (Goldreich, P. & Sridhar, S. 1995) which predicts a  $k^{-2}$  power-law for a field-aligned flows.

Which theory can describe a  $k^{-5/3}$  power-law exhibited by a field-aligned flows?

# NI MHD 2D + Slab spectral theory



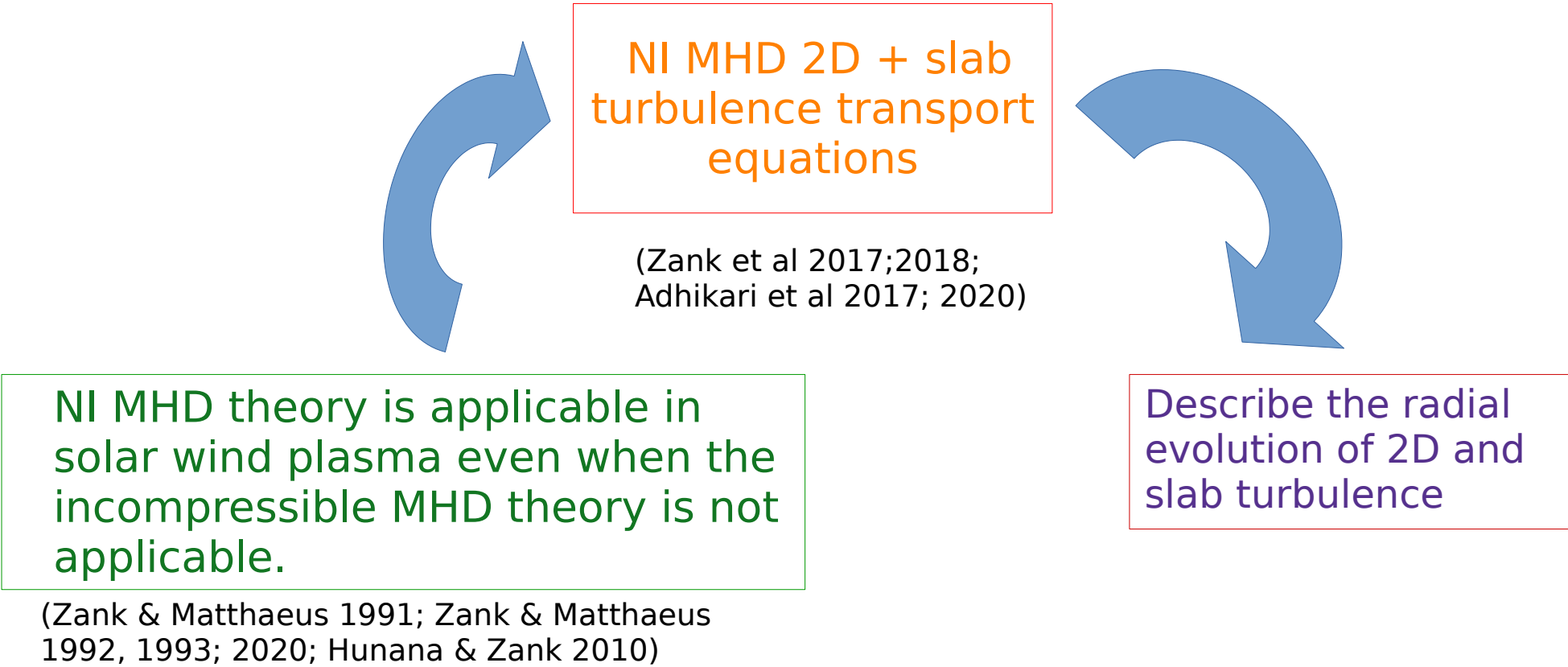
➤ For strongly imbalanced turbulence, we find

$$G^*(k_{\perp}) = \varepsilon_*^{1/2} \varepsilon_{\infty}^{1/6} k_{\perp}^{-5/3} \quad \text{and}$$

$$G^*(k_z) = \varepsilon_*^{1/2} \varepsilon_{\infty}^{1/6} k_t^{2(a-1)/3} k_z^{-(2a+3)/3},$$

➤ The choice  $a = 1$  corresponds to a parallel spectral index of  $-5/3$ , as we observe in the PSP data.

(Zank et al. 2020)



NI MHD 2D + slab  
turbulence transport  
equations

(Zank et al 2017;2018;  
Adhikari et al 2017; 2020)

NI MHD theory is applicable in  
solar wind plasma even when the  
incompressible MHD theory is not  
applicable.

(Zank & Matthaeus 1991; Zank & Matthaeus  
1992, 1993; 2020; Hunana & Zank 2010)

Describe the radial  
evolution of 2D and  
slab turbulence



# Transport of NI MHD turbulence (Zank et al 2017)

**2D**

$$\frac{\partial \mathbf{z}^{\infty \pm}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{z}^{\infty \pm} + \boxed{\mathbf{z}^{\infty \mp} \cdot \nabla \mathbf{z}^{\infty \pm}} + \mathbf{z}^{\infty \mp} \cdot \nabla \mathbf{U} + \frac{\mathbf{z}^{\infty \pm} - \mathbf{z}^{\infty \mp}}{4} \nabla \cdot \mathbf{U} - \frac{\mathbf{z}^{\infty \pm} - \mathbf{z}^{\infty \mp}}{4} \mathbf{z}^{\infty \mp} \frac{1}{\rho} \cdot \nabla \rho = -\frac{1}{\rho} \nabla \left( P^{\infty} + \frac{B^{\infty 2}}{2\mu_0} \right).$$

Nonlinear terms

→ No Alfvén wave effect

**Slab**

$$\frac{\partial \mathbf{z}^{*\pm}}{\partial t} + (\mathbf{U} \mp \mathbf{V}_{A0}) \cdot \nabla \mathbf{z}^{*\pm} + \boxed{\mathbf{z}^{\infty \mp} \cdot \nabla \mathbf{z}^{*\pm} + \mathbf{z}^{*\mp} \cdot \nabla \mathbf{z}^{\infty \pm} + \mathbf{z}^{*\mp} \cdot \nabla \mathbf{z}^{*\pm}} + \frac{1}{4} (\mathbf{z}^{*\pm} - \mathbf{z}^{*\mp}) \nabla \cdot \mathbf{U} + \mathbf{z}^{*\mp} \cdot \nabla (\mathbf{U} \pm \mathbf{V}_{A0}) \pm \frac{1}{2} \frac{1}{\rho} \mathbf{z}^{*\mp} \cdot \nabla \rho_{sw} \mathbf{V}_{A0} - \frac{1}{4} (\mathbf{z}^{*+} - \mathbf{z}^{*-}) \frac{1}{\rho} \mathbf{V}_{A0} \cdot \nabla \rho \pm \frac{1}{4} (\mathbf{z}^{\infty +} - \mathbf{z}^{\infty -}) \frac{1}{\rho} \mathbf{z}^{*\mp} \cdot \nabla \rho \mp \frac{1}{4} (\mathbf{z}^{*+} - \mathbf{z}^{*-}) \frac{1}{\rho} \mathbf{z}^{\infty \pm} \cdot \nabla \rho \mp \frac{1}{2} (\mathbf{z}^{*+} - \mathbf{z}^{*-}) \cdot \nabla \mathbf{U} = -\frac{1}{\rho} \nabla \left( P^* + \frac{1}{\mu_0} \mathbf{B}^* \cdot \mathbf{B} + \frac{1}{\mu_0} \mathbf{B}^* \cdot \mathbf{B}^{\infty} \right).$$

Nonlinear terms

→ Alfvén wave effect

$$\mathbf{z}^{\infty \pm} \equiv \mathbf{u}^{\infty} \pm \frac{\mathbf{B}^{\infty}}{\sqrt{\mu_0 \rho(\mathbf{x})}} \equiv \mathbf{u}^{\infty} \pm \mathbf{v}_A^{\infty},$$

$$\mathbf{z}^{*\pm} \equiv \mathbf{u}_1 \pm \frac{\mathbf{B}^*}{\sqrt{\mu_0 \rho(\mathbf{x})}} \equiv \mathbf{u}_1 \pm \mathbf{v}_A^*.$$

# NI MHD Turbulence Transport Equation

2D

$$\begin{aligned}
 U \frac{dE_T^\infty}{dr} = & - \left( \frac{E_T^\infty}{2} + \left( 2a - \frac{1}{2} \right) E_D^\infty \right) \frac{dU}{dr} - \frac{2U}{r} \left( \frac{E_T^\infty}{2} + \left( 2a - \frac{1}{2} \right) E_D^\infty \right) \\
 & - \frac{U}{\sigma f(r)} \left( \frac{E_T^\infty}{2} + \left( 2a - \frac{1}{2} \right) E_D^\infty \right) \exp \left( \frac{r - r_a}{\sigma} \right) \frac{f_m - f(r)}{\exp \left( \frac{r - r_a}{\sigma} \right) + 1} \\
 & - \alpha \frac{|E_T^\infty + E_C^\infty|^2 |E_T^\infty - E_C^\infty|^{1/2}}{L_\infty^+} - \alpha \frac{|E_T^\infty - E_C^\infty|^2 |E_T^\infty + E_C^\infty|^{1/2}}{L_\infty^+} \\
 & + \frac{S^{\langle z^{\infty+2} \rangle} + S^{\langle z^{\infty-2} \rangle}}{2};
 \end{aligned}$$

← Superradial expansion

← Nonlinear term

← Turbulence source

NI/slab

$$\begin{aligned}
 (U - V_A) \frac{dE_T^*}{dr} = & - \frac{1}{2} \frac{dU}{dr} E_T^* + (2b - 1) \frac{U}{r} E_T^* + \frac{V_A}{2\rho} \frac{d\rho}{dr} E_T^* \\
 & + \frac{1}{2} (2b - 1) \frac{U}{\sigma f(r)} E_T^* \exp \left( \frac{r - r_a}{\sigma} \right) \frac{f_m - f(r)}{\exp \left( \frac{r - r_a}{\sigma} \right) + 1} \\
 & - 2\alpha \frac{E_T^* |E_T^\infty + E_C^\infty| |E_T^\infty - E_C^\infty|^{1/2}}{L_\infty^+} \\
 & + \frac{S^{\langle z^{*+2} \rangle}}{2},
 \end{aligned}$$

(Zank et al 2017, 2018; Adhikari et al 2020,2022, Telloni et al 2022)

# Conservation of Turbulence Energy

2D

$$\frac{1}{r^2 f(r)} \frac{d}{dr} \left[ r^2 f(r) U (E_w^\infty + P_w^\infty) \right] = U \frac{dP_w^\infty}{dr} + \rho \left[ -\alpha \frac{|E_T^\infty + E_C^\infty|^2 |E_T^\infty - E_C^\infty|^{1/2}}{L_\infty^+} - \alpha \frac{|E_T^\infty - E_C^\infty|^2 |E_T^\infty + E_C^\infty|^{1/2}}{L_\infty^+} + \frac{S^{\langle z^{\infty+2} \rangle} + S^{\langle z^{\infty-2} \rangle}}{2} \right],$$

Turbulence energy density

$$E_w^\infty = \rho E_T^\infty / 2$$

Turbulence pressure

$$P_w^\infty = \frac{\rho}{2} \left[ \frac{E_T^\infty}{2} + \left( 2a - \frac{1}{2} \right) E_D^\infty \right] = \frac{E_w^\infty}{2} \left[ 1 + 2 \left( 2a - \frac{1}{2} \right) \sigma_D^\infty \right]$$

NI/slab

$$\frac{1}{r^2 f(r)} \frac{d}{dr} \left[ r^2 f(r) ((U - V_A) E_w^* + U P_w^*) \right] = U \frac{dP_w^*}{dr} + 2E_w^* \left( 4b \frac{u}{r} + 2b \frac{U}{f(r)} \frac{df(r)}{dr} \right) + \frac{\rho}{2} \left[ -2\alpha \frac{E_T^* |E_T^\infty + E_C^\infty| |E_T^\infty - E_C^\infty|^{1/2}}{L_\infty^+} + \frac{S^{\langle z^{*+2} \rangle}}{2} \right],$$

Turbulence energy density

$$E_w^* = \rho E_T^* / 2$$

Turbulence pressure

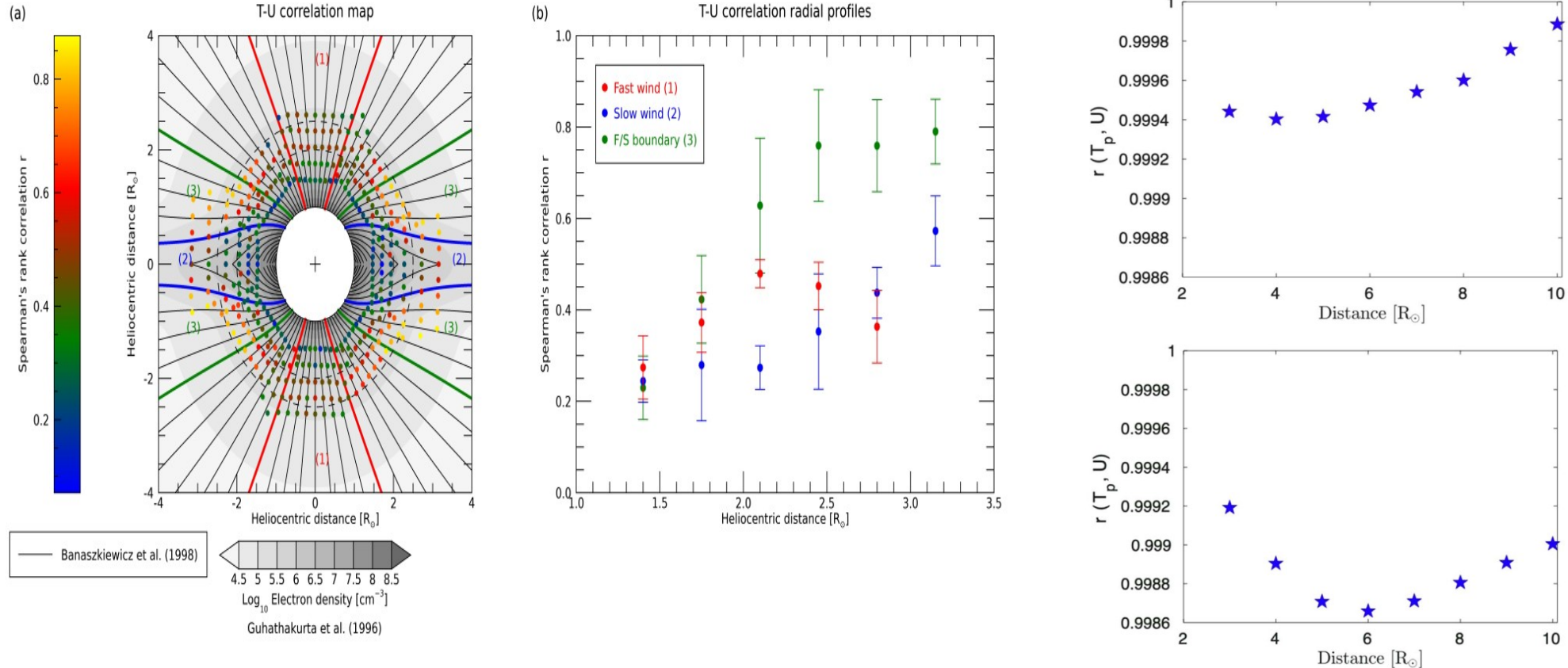
$$P_w^* = E_w^* / 2$$

(Adhikari et al 2022, see also Wang et al. 2022)



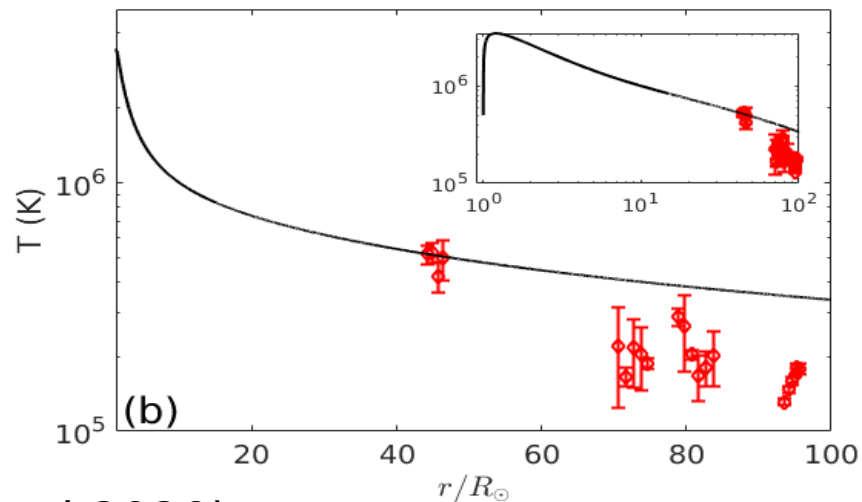
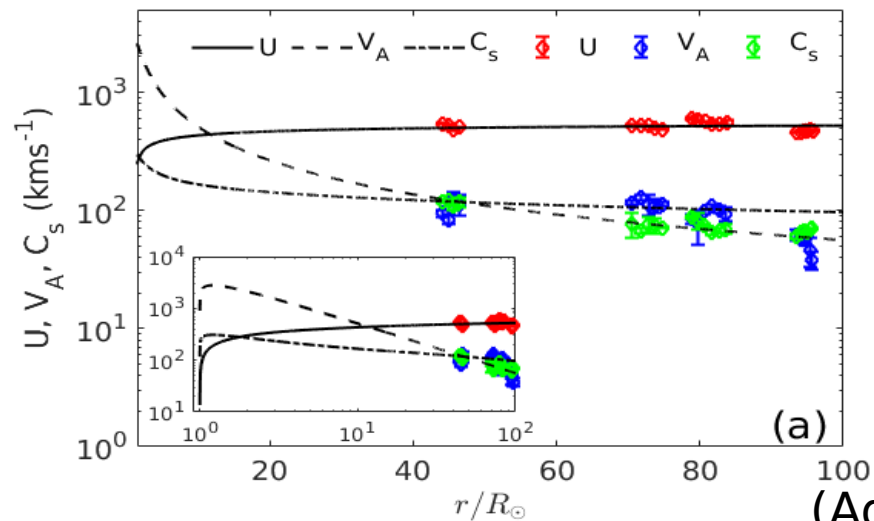
# Possible Evidence for Shear-driven Kelvin–Helmholtz Instability along the Boundary of Fast and Slow Solar Wind in the Corona

Daniele Telloni<sup>1</sup> , Laxman Adhikari<sup>2</sup> , Gary P. Zank<sup>2,3</sup> , Lingling Zhao<sup>2</sup> , Luca Sorriso-Valvo<sup>4,5</sup> , Ester Antonucci<sup>1</sup> , Silvio Giordano<sup>1</sup> , and Salvatore Mancuso<sup>1</sup>

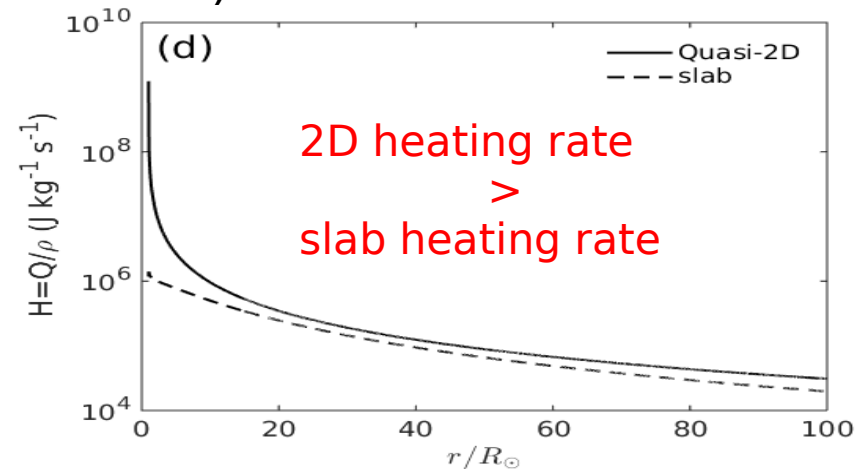
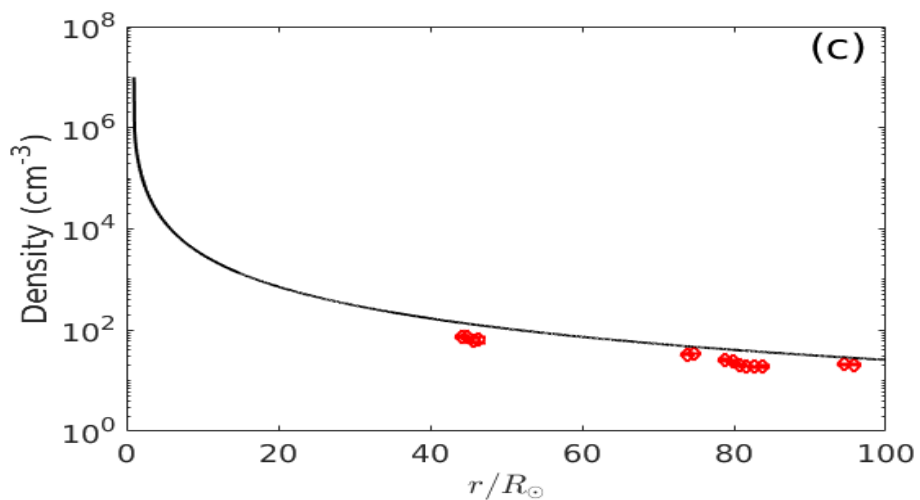


Model results + Fast solar wind

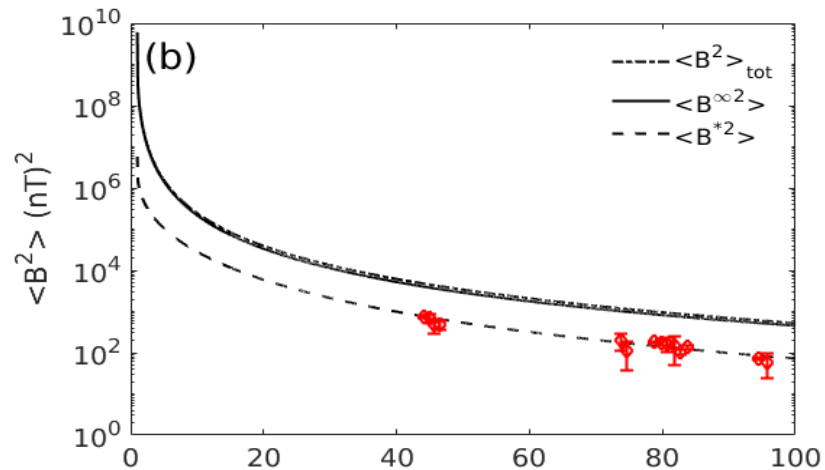
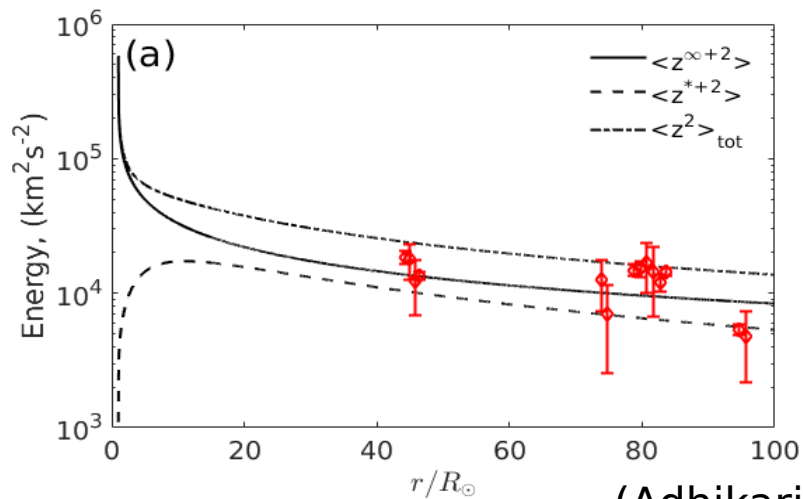
# Model results + PSP data



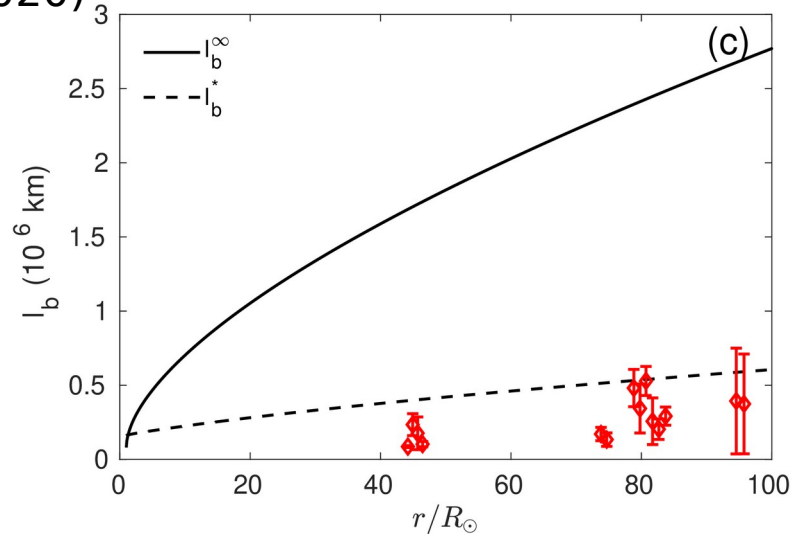
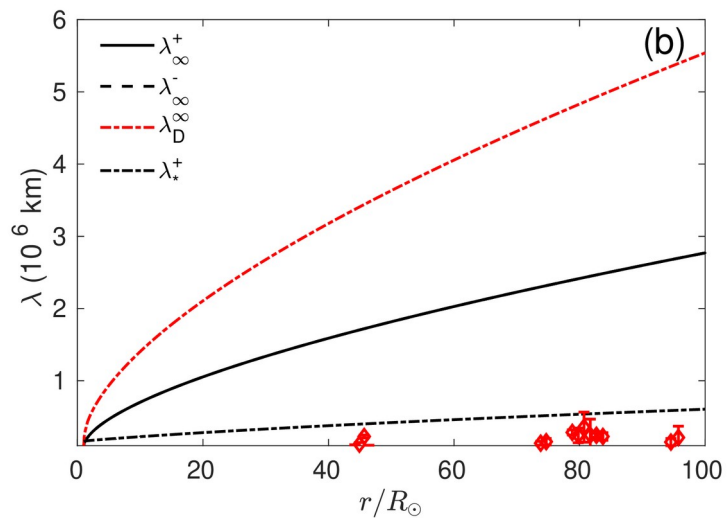
(Adhikari et al 2020)



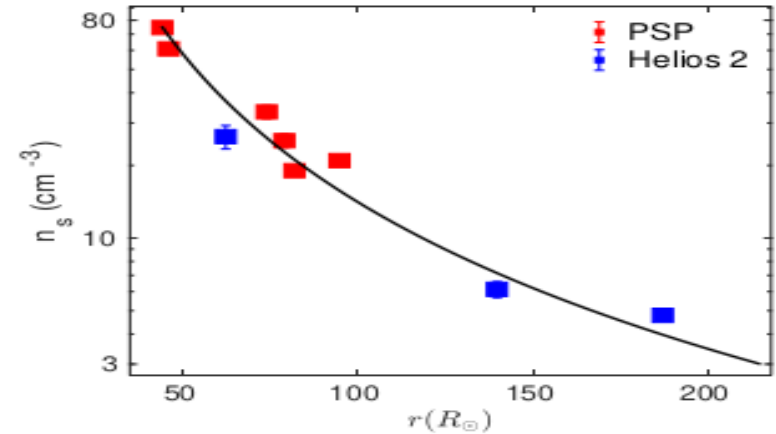
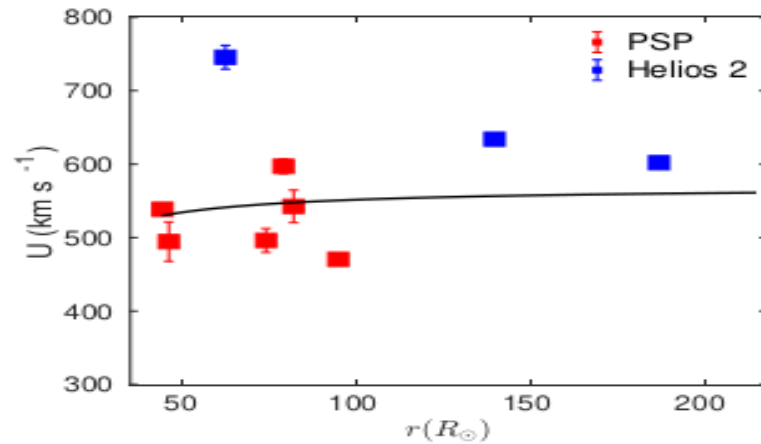
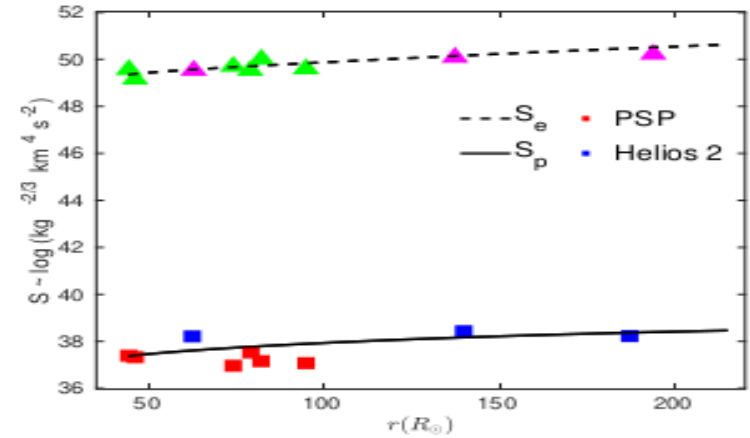
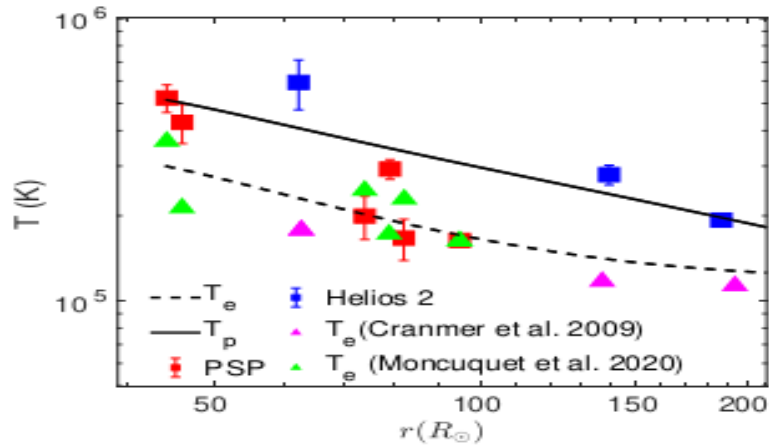
# Model results + PSP data



(Adhikari et al 2020)



# Model results + PSP & Helios 2 data



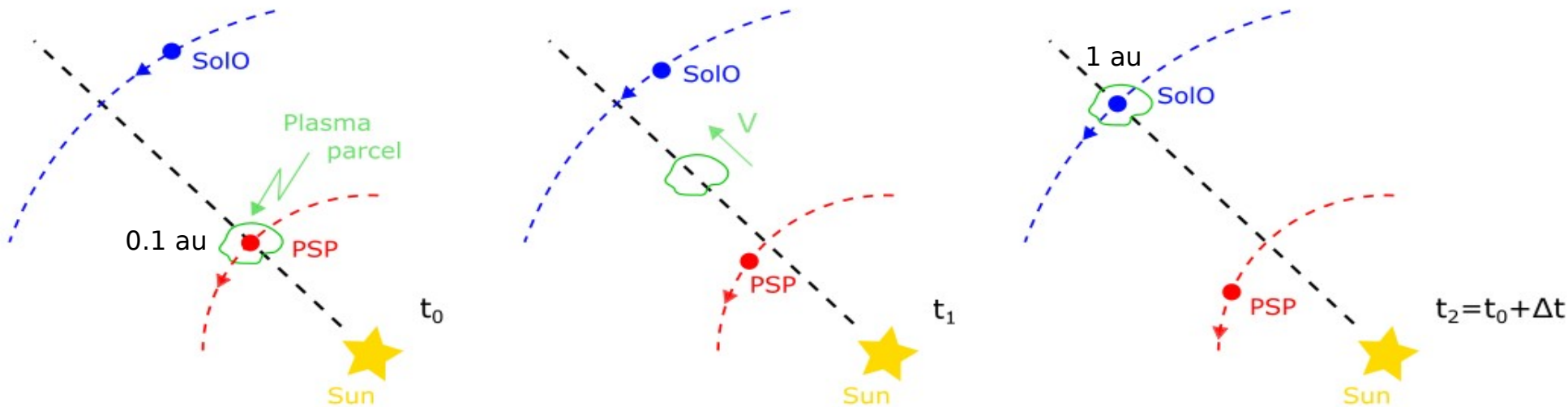
Electron's effect is included.

(Adhikari et al 2021)



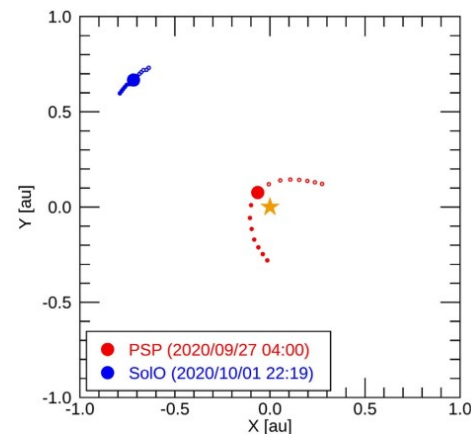
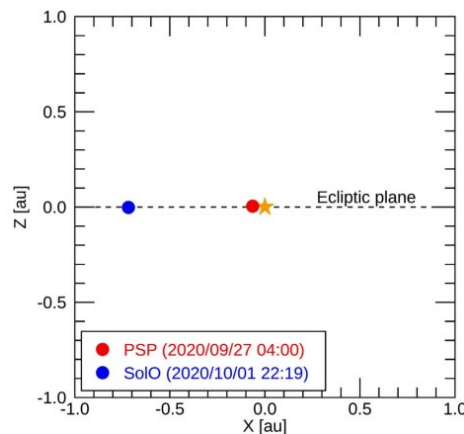
Model results + Slow solar wind

# First Parker Solar Probe–Solar Orbiter Radial Alignment

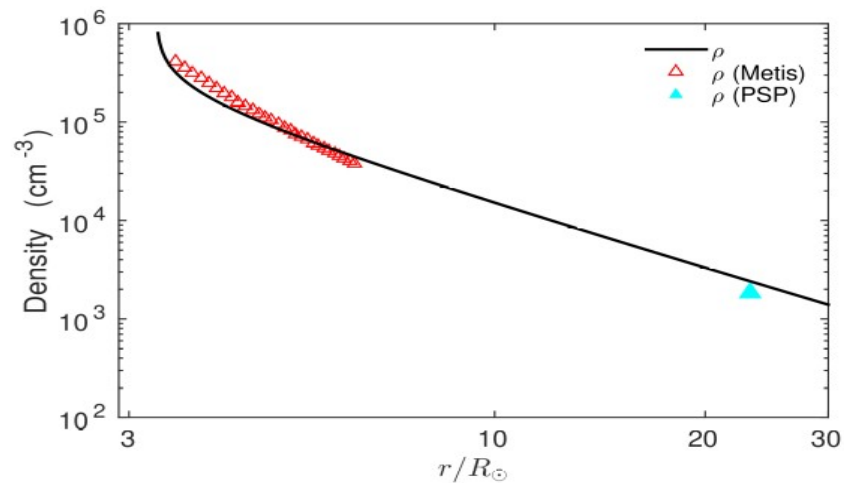
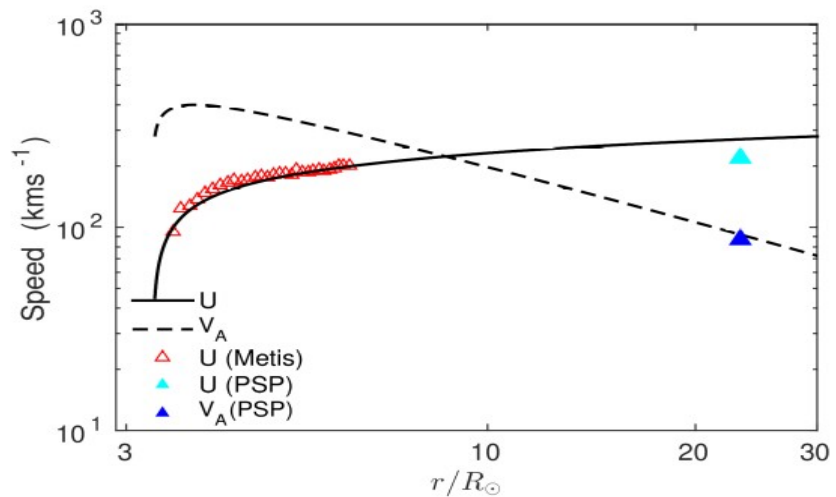


- The identification of the time intervals corresponding to the same plasma parcel observed at PSP and SolO distances during their radial alignment with the Sun is based on the ballistic approach.

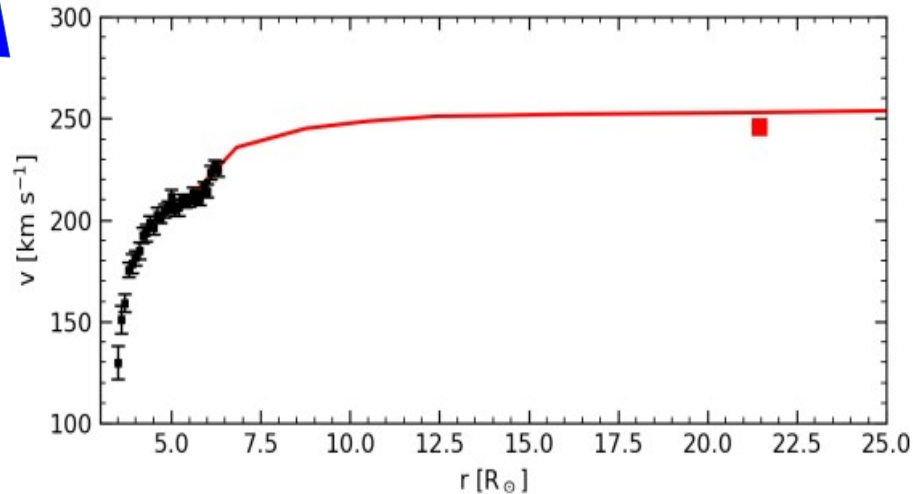
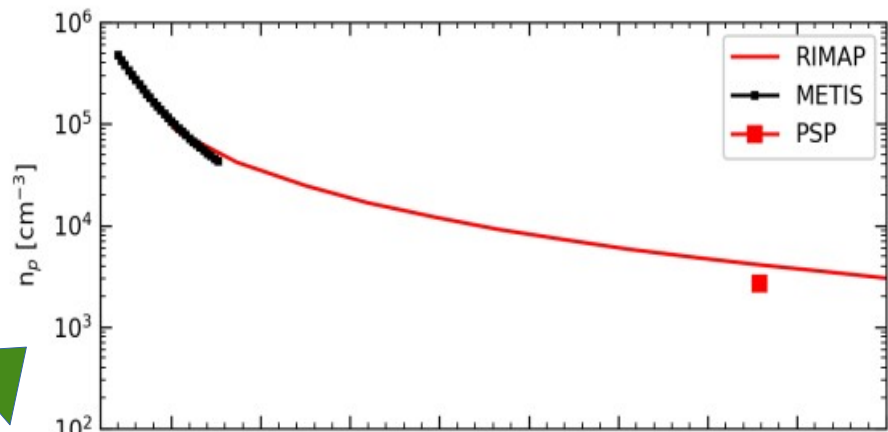
(Telloni et al 2021)



# Evolution of Slow Solar Wind



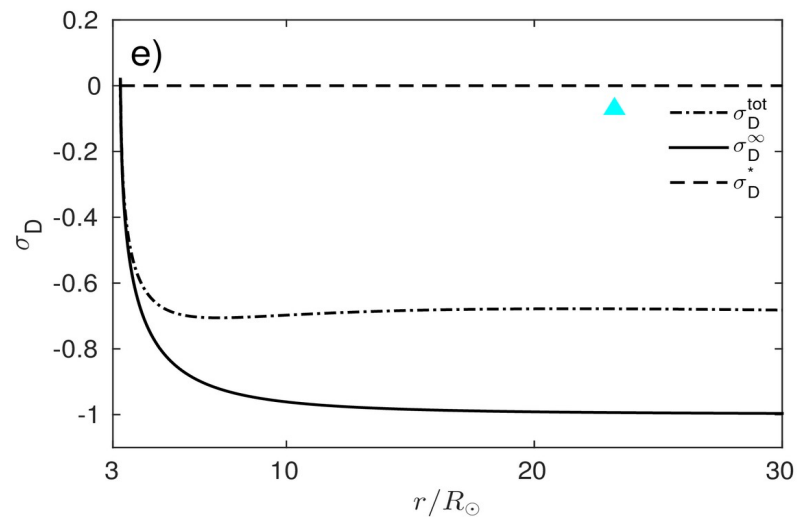
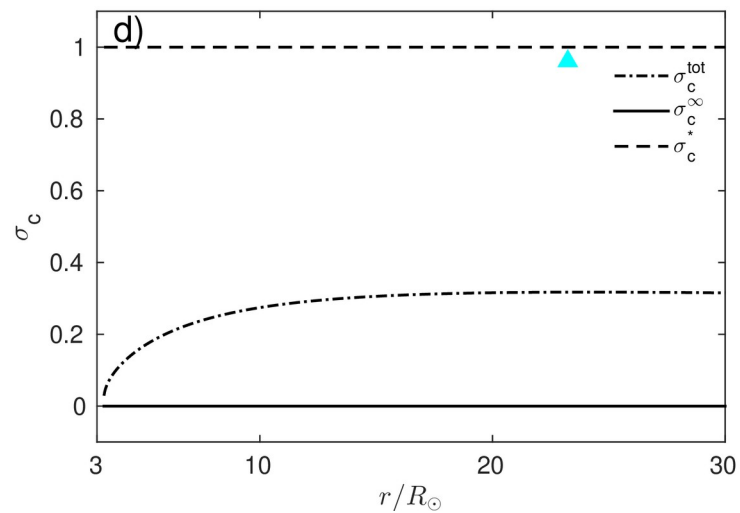
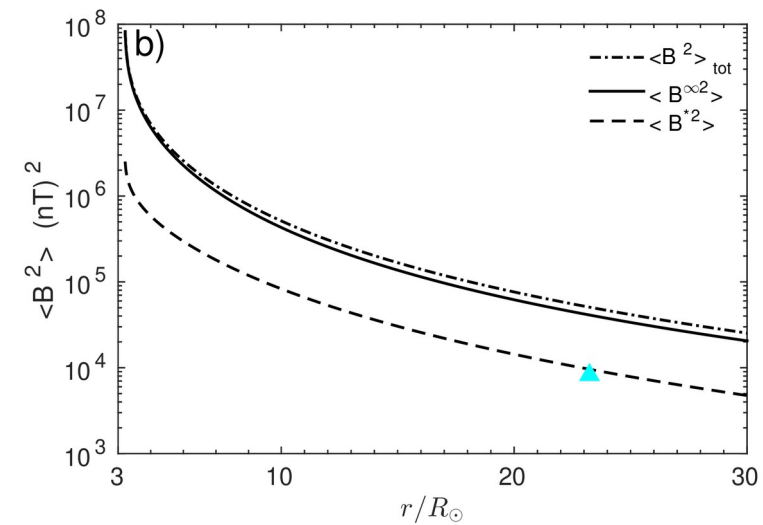
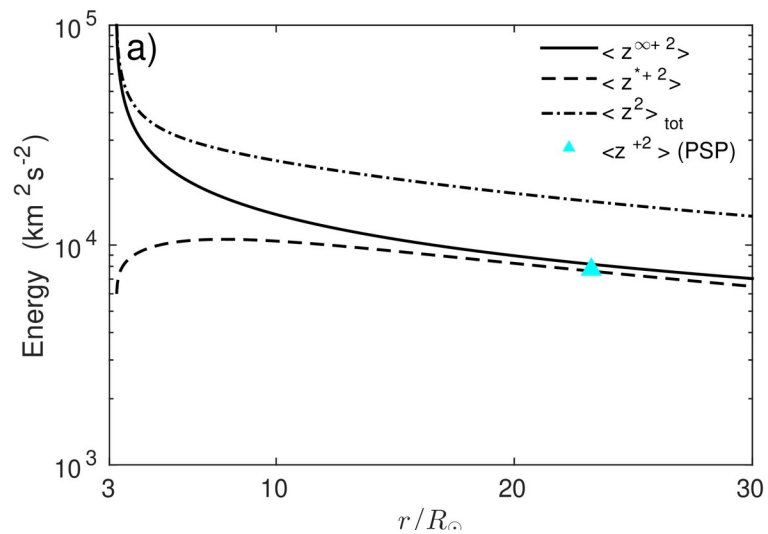
(Adhikari et al 2022)



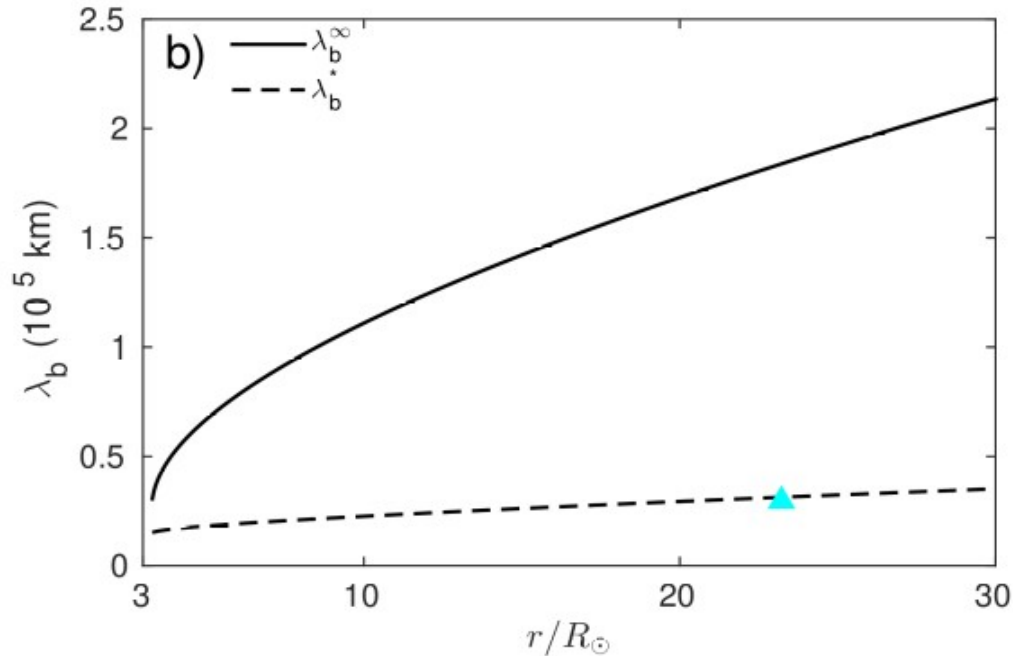
(Biondo et al 2022)

# Evolution of Turbulence Energy

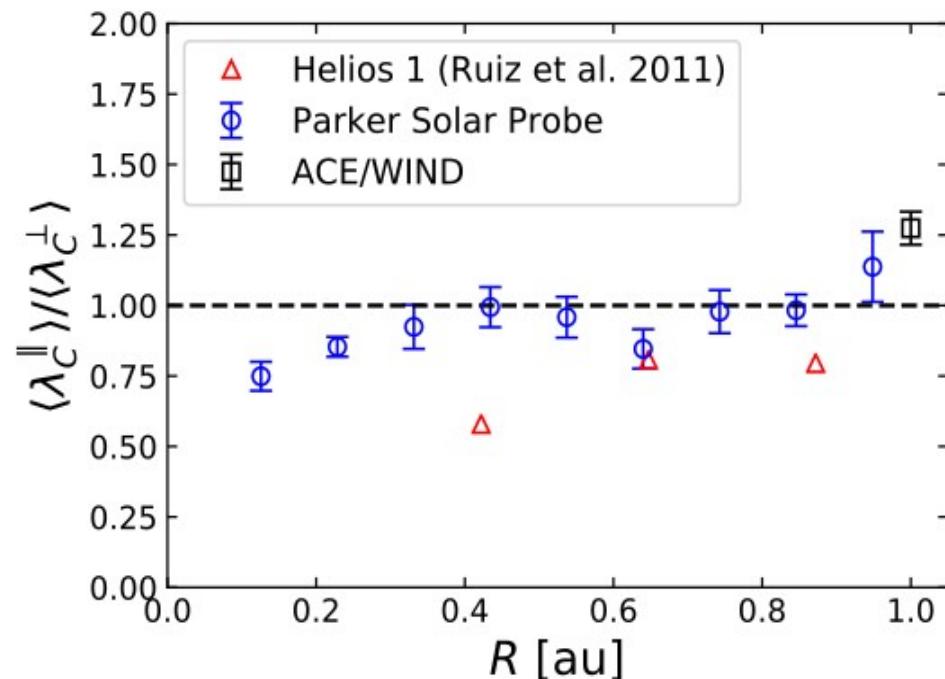
(Adhikari et al 2022)



# Evolution of Correlation Length



(Adhikari et al 2022)



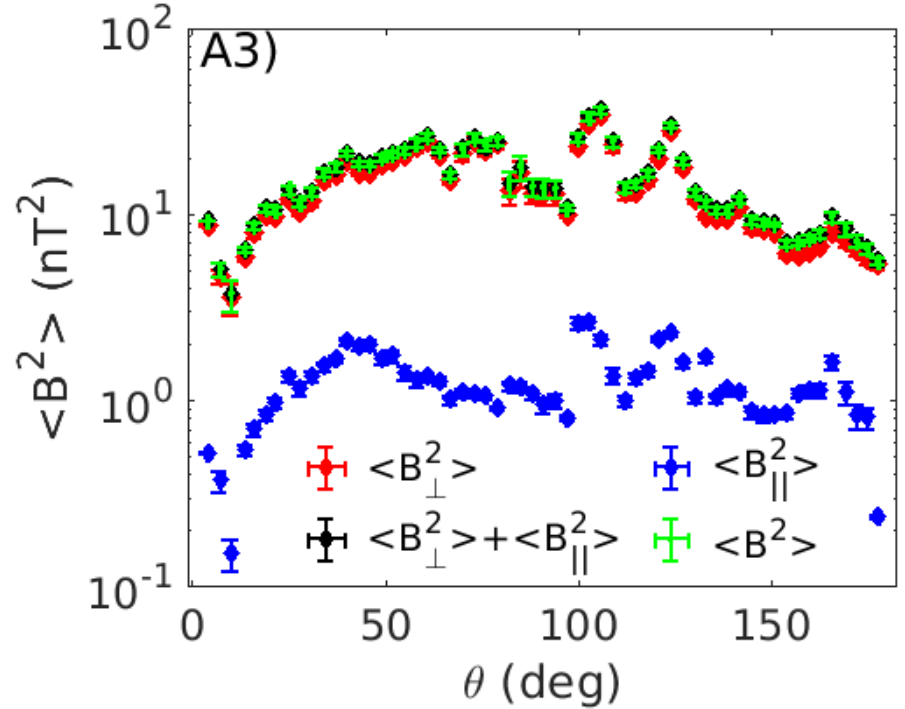
(Cuesta et al 2022)

The theoretical 2D correlation length (solid curves) increases much more rapidly than the theoretical NI/slab correlation length. This is different from Ruiz et al. (2011) and Cuesta et al. (2022). In their results, the slab (or parallel) correlation length increases faster than the 2D (or perpendicular) correlation length.

# Turbulence energy vs Angle

$$f P_{SUM}(f) = \underbrace{2 C_S \left( \frac{2\pi f}{V_w \cos \psi} \right)^{1-q}}_{\text{Slab}} + \underbrace{2 C_2 \left( \frac{2\pi f}{V_w \sin \psi} \right)^{1-q}}_{\text{2D}}$$

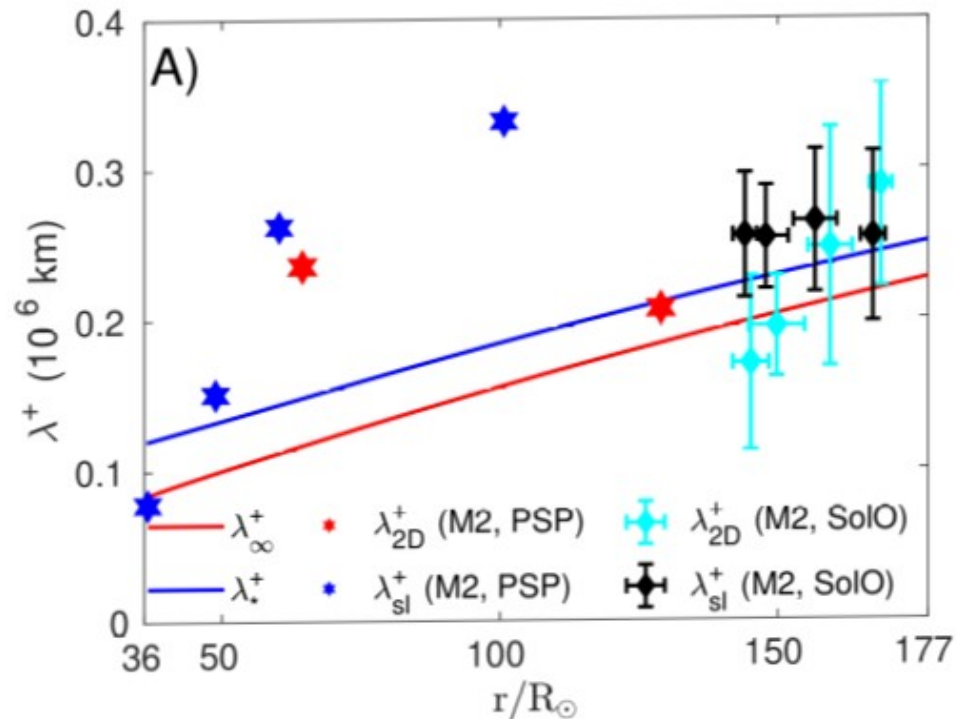
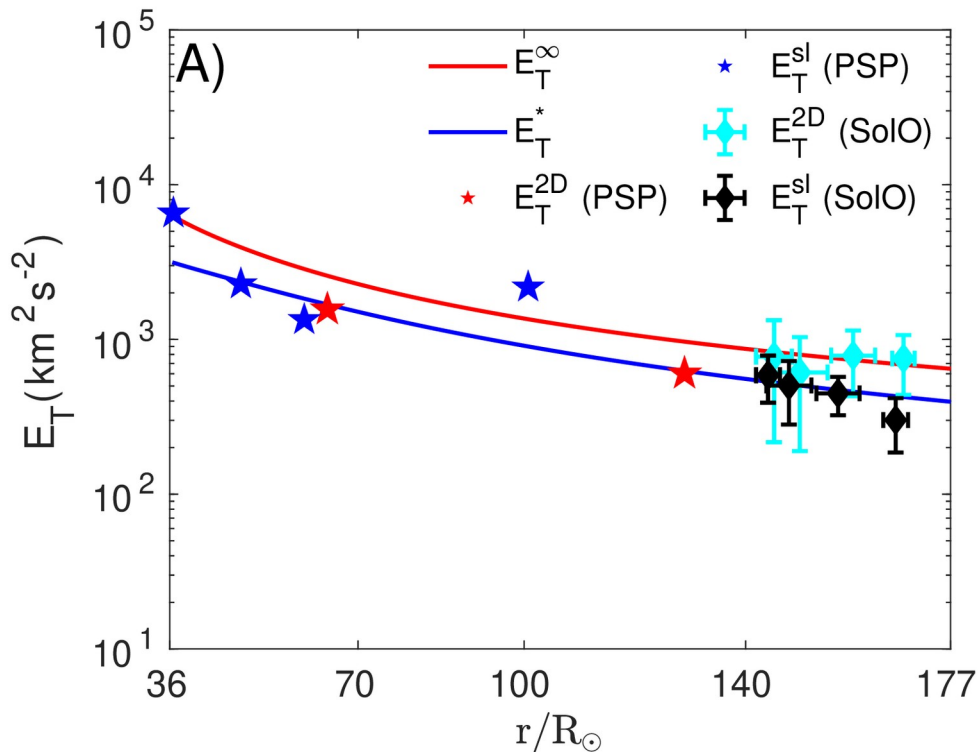
(Bieber et al 1996)



(Adhikari et al 2021)

# 2D and slab turbulence energy

(Adhikari et al 2022)



i)  $0^\circ < \theta_{UB} < 25^\circ$  and  $155^\circ < \theta_{UB} < 180^\circ$ , and ii)

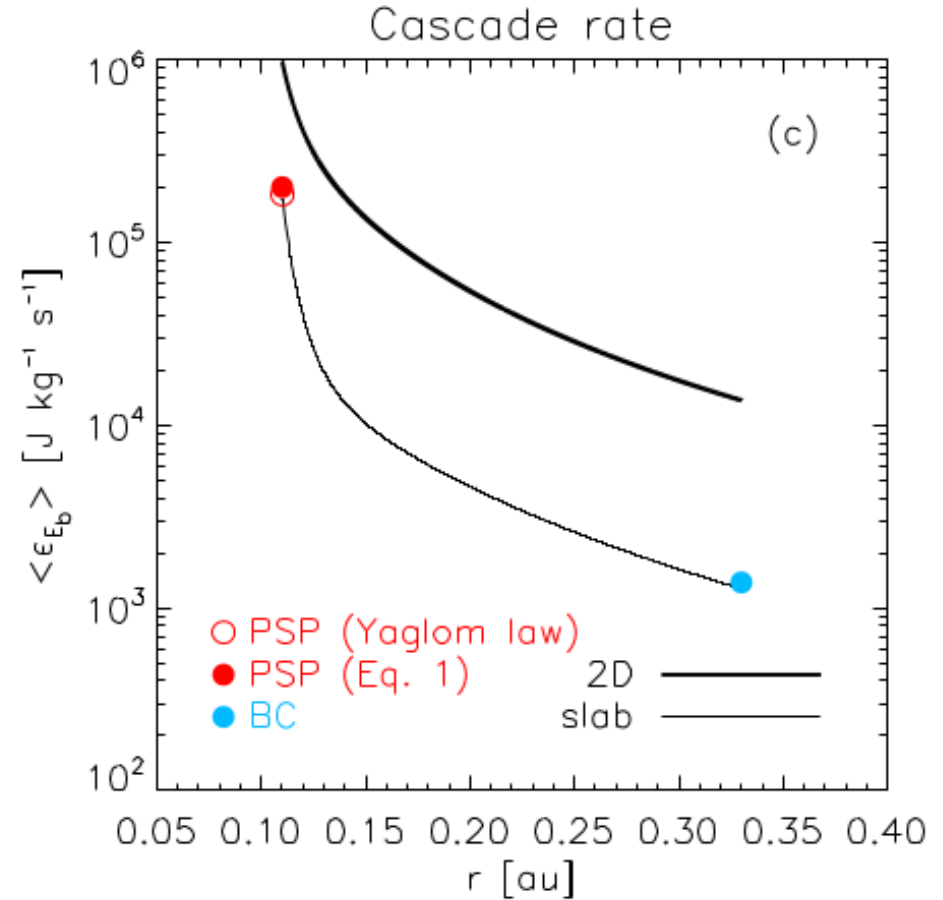
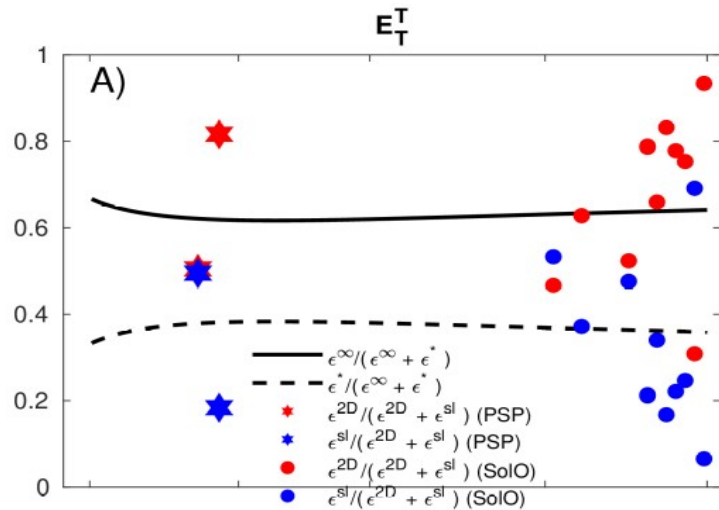
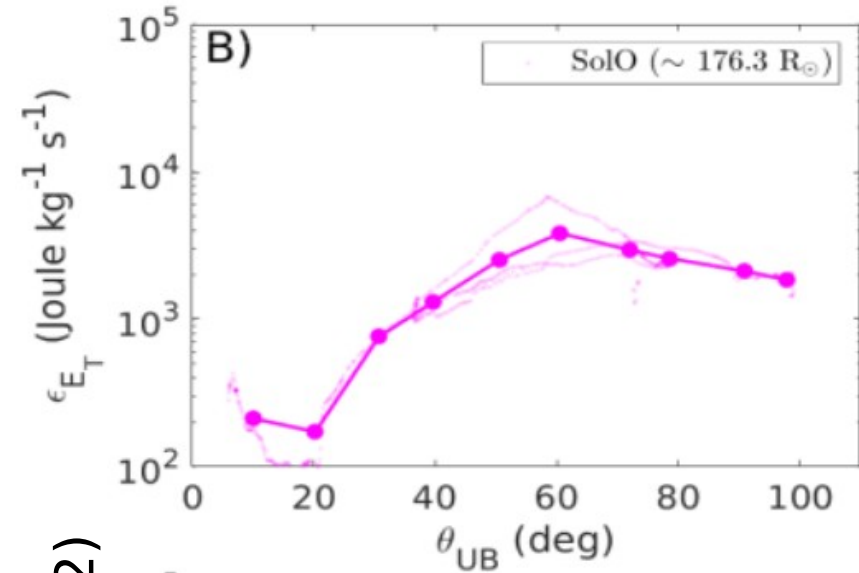
Slab turbulence

$65^\circ < \theta_{UB} < 115^\circ$

2D turbulence

# 2D and slab cascade rate

(Adhikari et al 2022)



(Telloni et al 2022)



# Summary

- Dissipation of 2D turbulence is mainly responsible for the heating of the solar corona and the acceleration of the solar wind.
- PSP (and Helios) observations of **large-scale SW and turbulence quantities consistent with NI/slab results** from quasi-2D model since the fast wind observations made in the fast field-aligned solar wind flow (only slab fluctuations visible to PSP).
- PSP observations of large-scale **Alfvenic slow SW and turbulence quantities consistent with NI/slab results**.
- Near the Sun, **PSP observes mainly the slab component**. Whereas, **Solo** observes both the 2D and slab components frequently.
- **2D turbulence energy/heating rate is larger than the slab turbulence energy/heating rate**.