Magneto-hydro-static computations in the quiet Sun

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Magneto-Hydro-Statics (MHS)

$$\mathbf{j} \times \mathbf{B} = \nabla P + \rho \nabla \Psi$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}, \; \nabla \cdot \mathbf{B} = 0$$

- ✓ MHS is a special class of MHD in equilibrium where the Lorentz-force is compensated by the pressure gradient force and gravity force.
- ✓ In the generic form the equations are nonlinear and even numerically difficult to solve. Several algorithms have been developed.
- ✓ High resolution vector magnetograms are required as boundary condition, which are available only for a few cases in active regions.
- ✓ Special classes of solutions allow a linearization of the MHS equations => Loss of generality, but easier to solve.

Linearized MHS-equations (Low 1991)

$$\underbrace{\nabla \times \mathbf{B} = \alpha \mathbf{B}}_{\text{Linear force-free part}} + \underbrace{a \exp(-\kappa z) \nabla B_z \times \mathbf{e_z}}_{\text{Nonmagnetic forces decrease with height}}$$

Linear force-free part

Nonmagnetic forces decrease with height

- ✓Linear MHS-equation can be solved with Fast Fourier Transform.
- ✓Require only line-of-sight magnetic field as boundary condition.
- ✓The solution has three free parameters: alpha, a, kappa
- ✓In active regions we get free parameters from vector magnetograms.
- ✓As vector magnetograms are not available In the quiet Sun => search for the optimum set of parameters by comparing MHS-solutions with observations.

How to obtain free parameters in Active regions?
=> use vector magnetograms

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} + a \exp(-\kappa z) \nabla B_z \times \mathbf{e_z}$$

$$\alpha = \frac{\sum \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \mathrm{sign}(B_z)}{\sum |B_z|}$$

Formula for linear force-free fields
(Hagino & Sakurai, 2004)

$$\oint T \mathbf{ds}$$

Write Maxwell Stress Tensor
(Forces) in components
Wiegelmann et al. 2017)

$$a = \frac{\left|\sum B_x B_z\right| + \left|\sum B_y B_z\right| + \left|\sum (B_x^2 + B_y^2) - B_z^2\right|}{\frac{1}{2}\sum (B_x^2 + B_y^2 + B_z^2)}$$

MHS-model in quiet Sun

- Aim: Compute an MHS-model from line-of-sight magnetogram and coronal (or chromospheric) images.
- Basic assumption: Magnetic field lines outline coronal loops
- Requirements:
  - Need Magnetogram and coronal image in same FOV (only part overlap is also ok)
  - A magnetic field model with free parameter(s)
  - Quantitative criteria to evaluate how well magnetic field model and image match.
- Heritage: Linear force-free model by Carcedo et al. 2003

Revisiting the Carcedo et al. 2003 approach

i) Align the magnetogram and the coronal image;

ii) Identify a coronal loop and its footpoint areas;

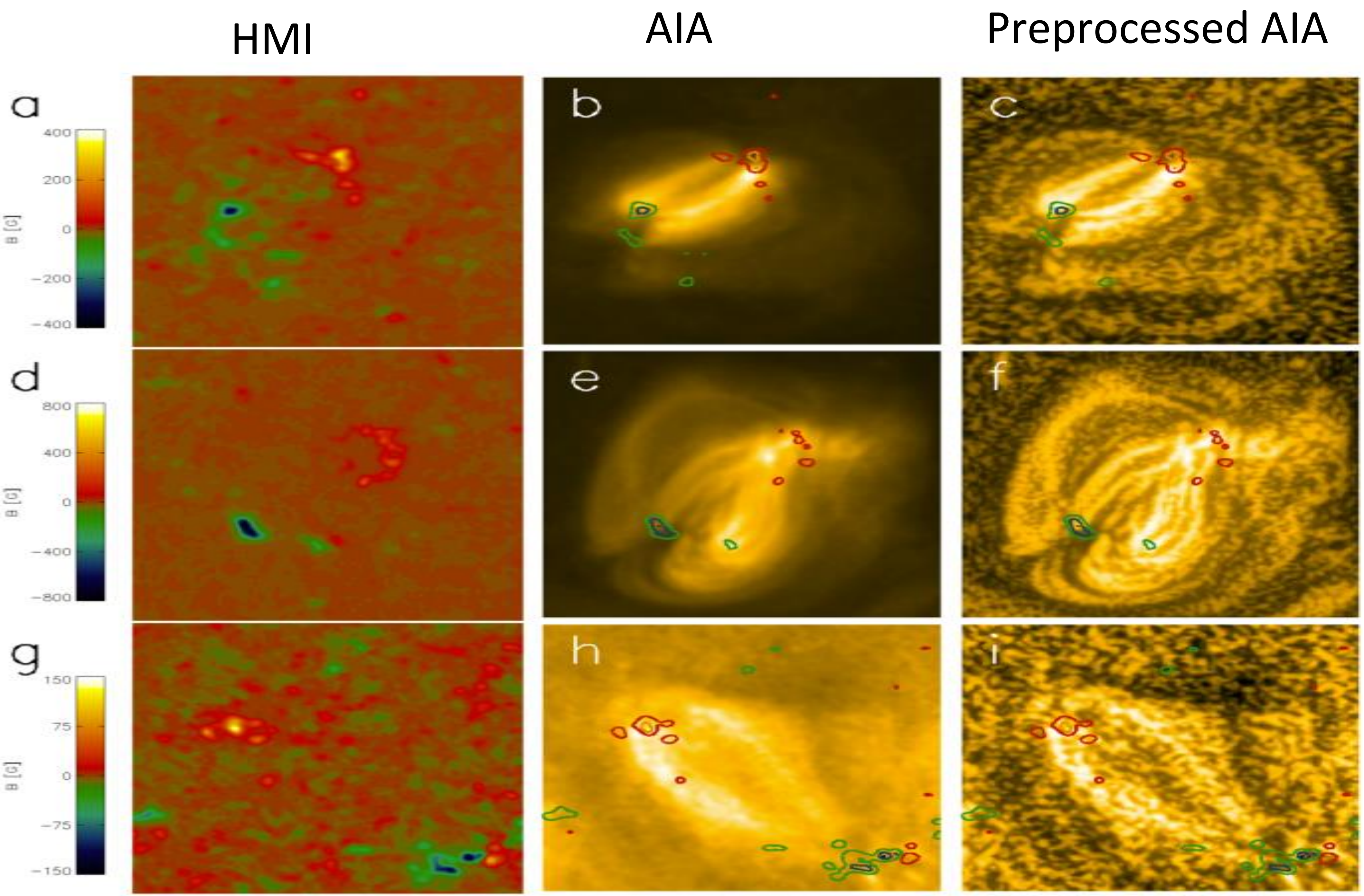
iii) Compute an LFF model with an arbitrary value of α;

iv) Compute a number of field lines starting from locations in the two footpoint areas. Only magnetic-field lines which connect both footpoint areas are further considered.

v) Each of these selected field lines is then quantitatively compared with the coronal image. A Gaussfit is applied to determine how far the centre of the loop (highest brightness in the intensity profile of the coronal loop) and the magnetic-field line are apart. This comparison is done at M positions along the field line.

vi) Compute the standard deviation $C_i(\alpha)$, which tells how well the LFF model with a certain parameter α agrees with a loop i seen in the image. This can be done by averaging over different magnetic-field lines or by taking the best fitting field line with minimum $C_i(\alpha)$.

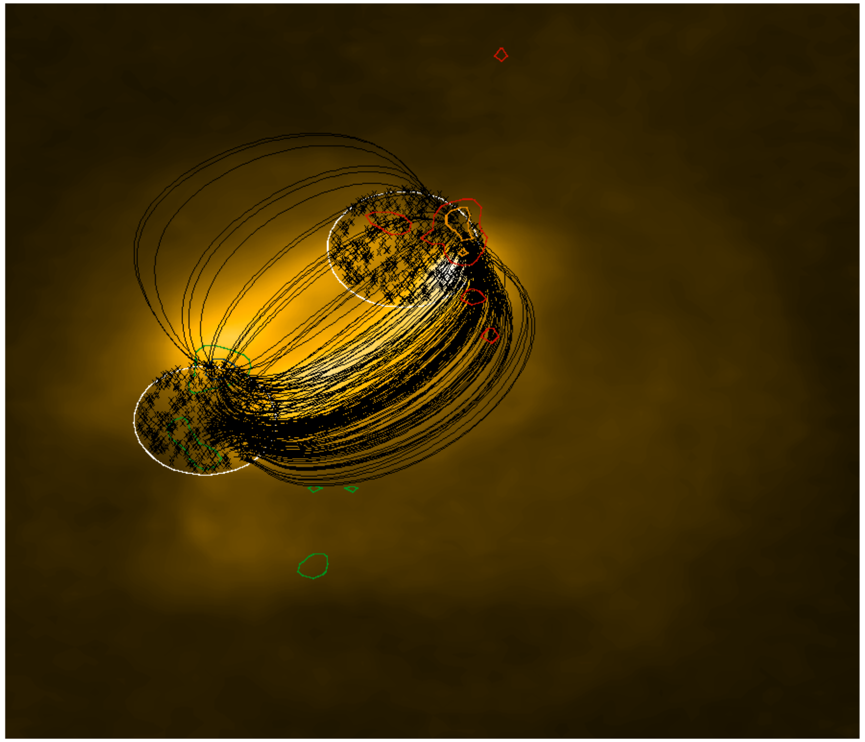
vii) The procedure is repeated for many values of α, thereby scanning the whole parameter space of the LFF model. The value of α with the minimum $C(\alpha)$ in step vi) is theoptimum and defines the best-fitting LFF model.

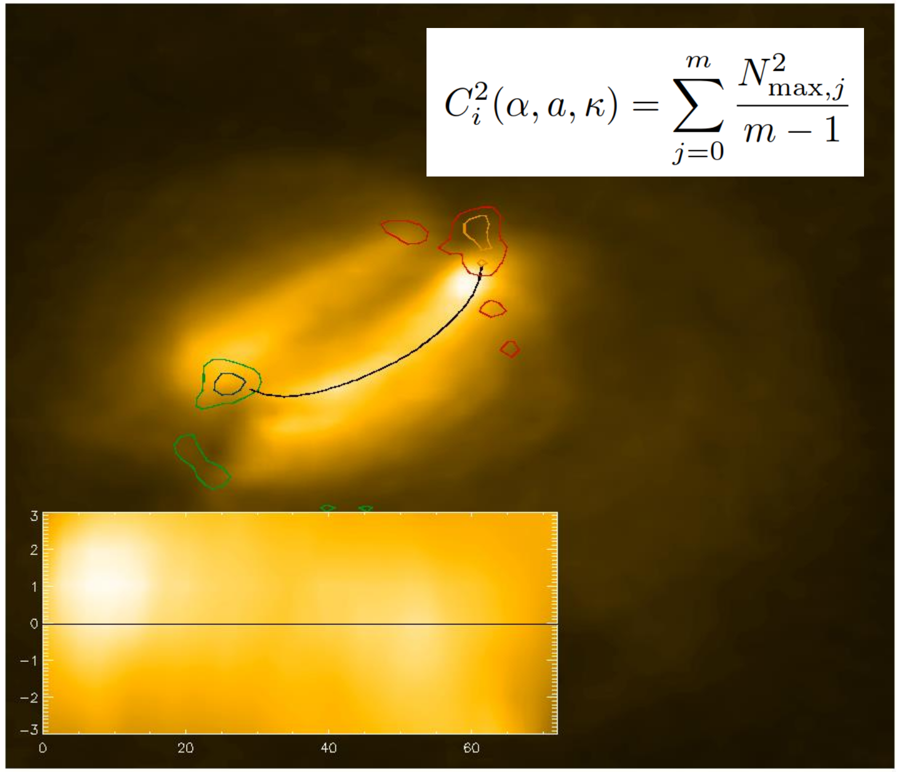


HMI
AIA
Preprocessed AIA
a
b
c
d
e
f
g
h
i
B [G]
400
200
0
−200
−400
800
400
0
−400
−800
150
75
0
−75
−150

How to compare magnetic field model and images?

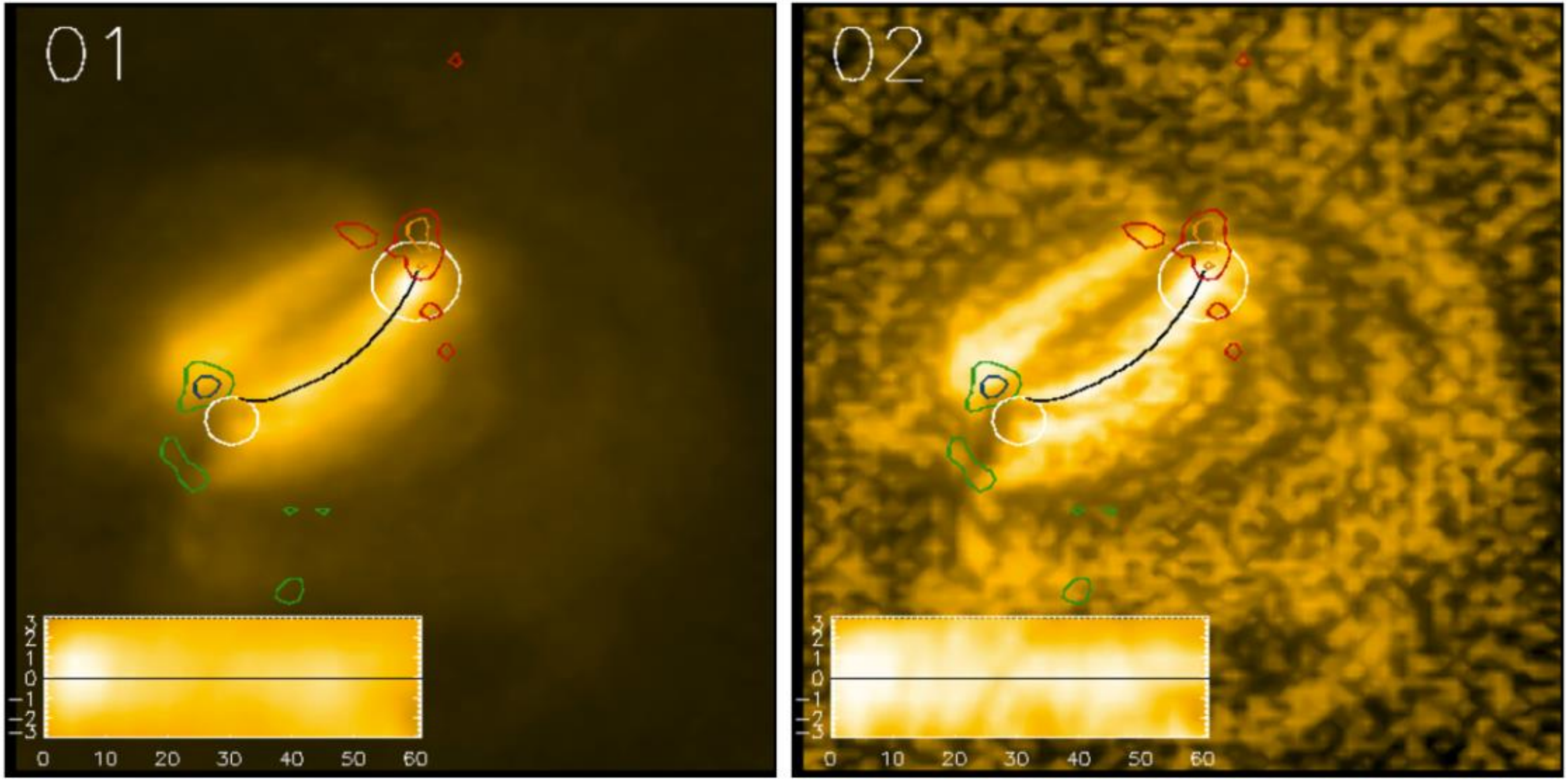
Modified version from Carcedo et. al. 2003



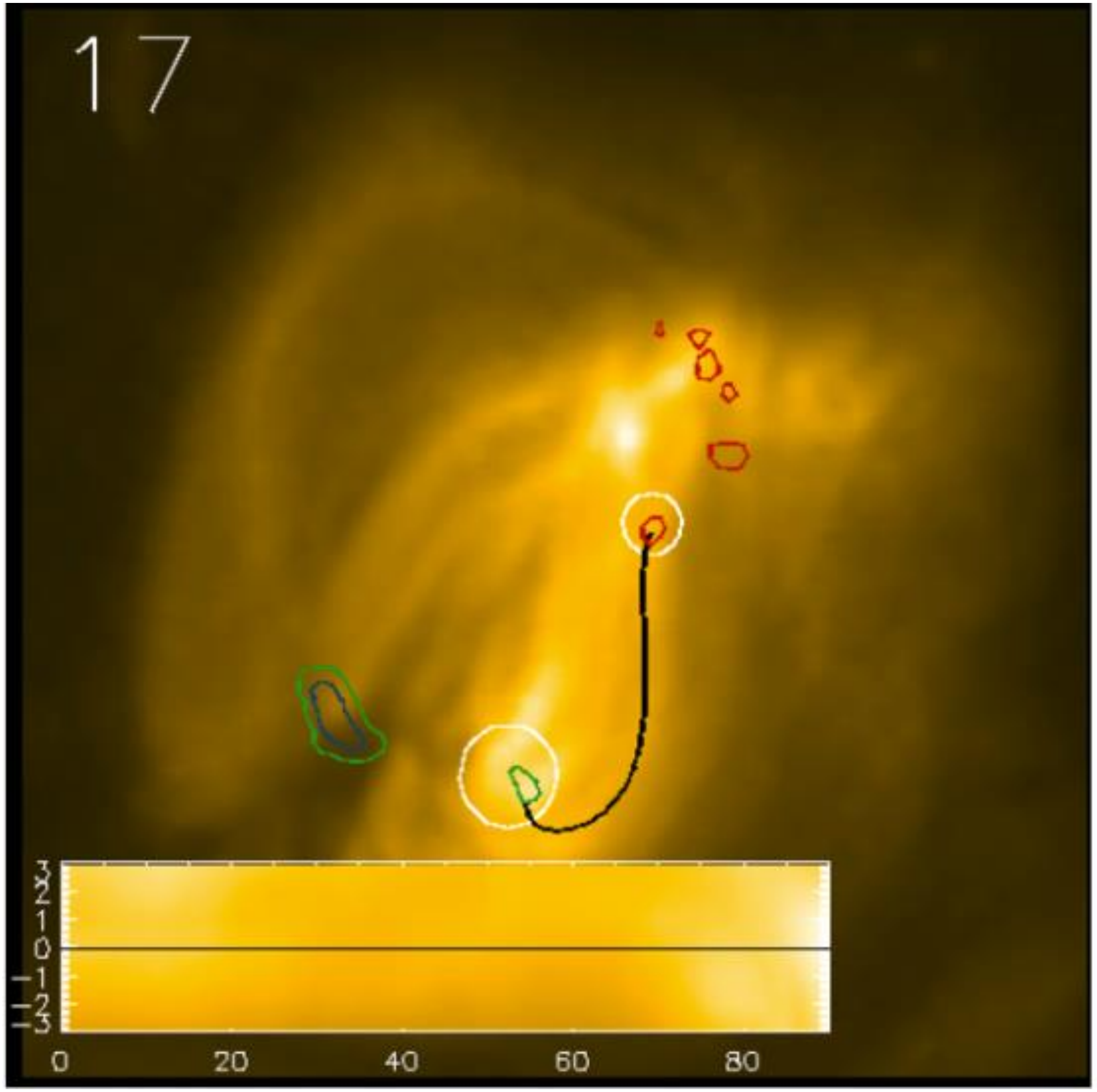


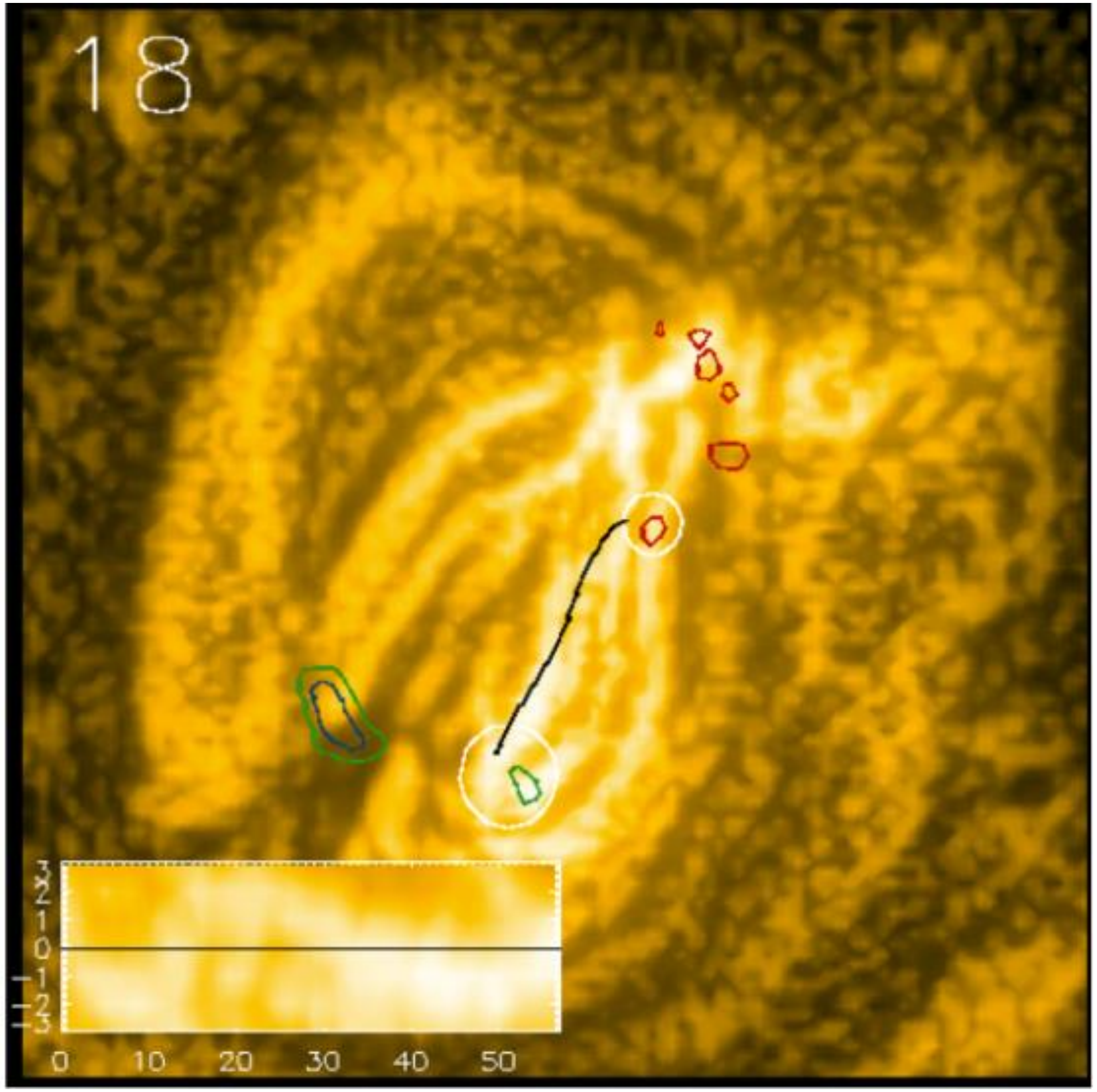
- ✓ Compute many loops from footpoint areas (white circles) of coronal loops.
- ✓ Consider only closed loops connecting the footpoints.
- ✓ Uncurl the loops and use Gaussfit to quantify how well a field line and a coronal loop agree.
- ✓ Find minimum with Simplex-Downhill iteration.

Uncurling Field lines

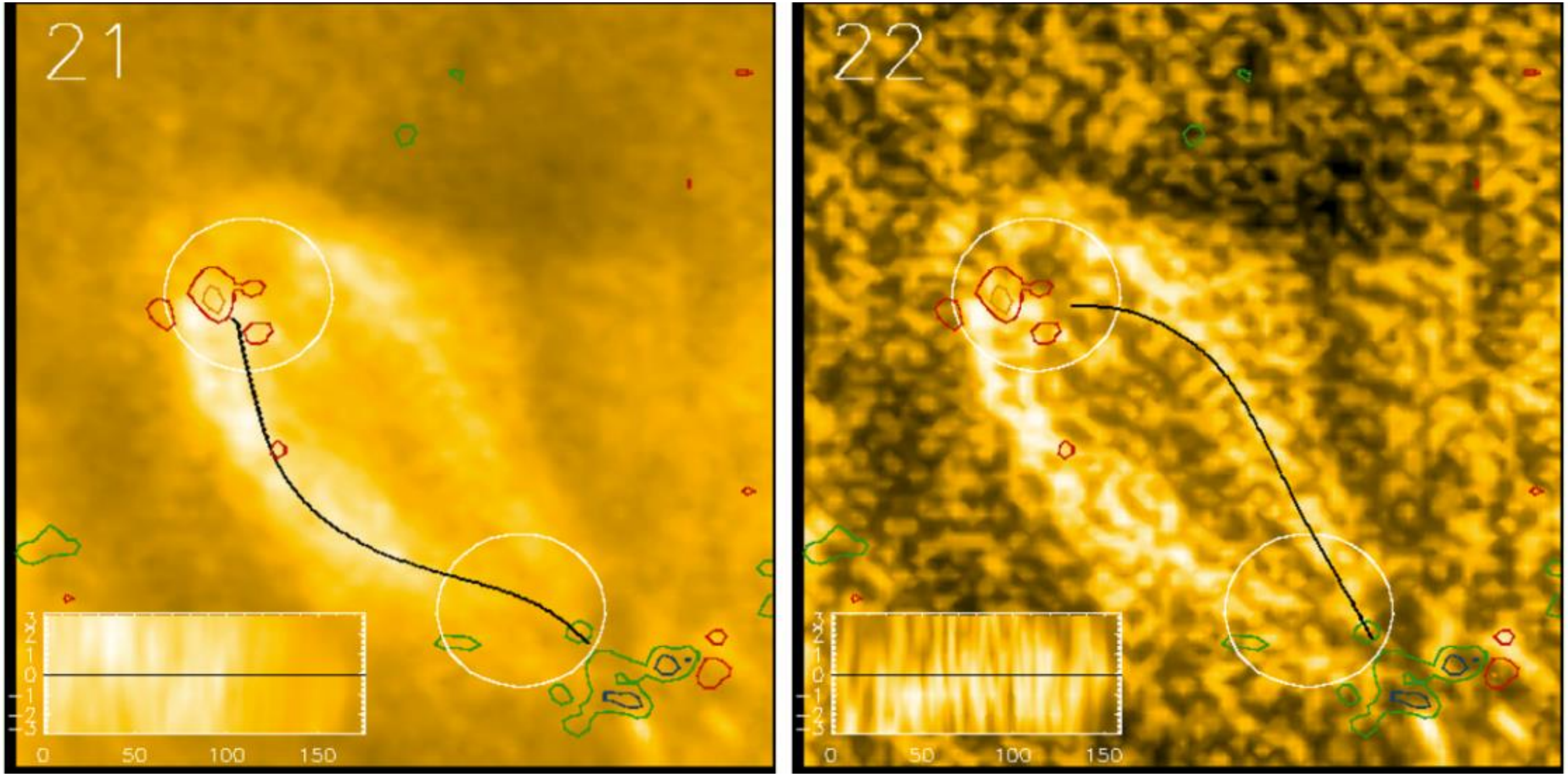


Uncurling Field lines





The uncurling method might identify different coronal structures dependent on choosen parameters



Case	n	L_{Pot}	L_{LFF}	α_{LFF}	L_{MHS}	α	a	κ
01 orig CBP-1	0	211	0.81	1.67	0.73	1.55	0.58	0.69
02 prep CBP-1	0	308	0.58	1.60	0.44	1.54	0.99	0.79
03 orig CBP-1	1	71	0.21	1.67	0.18	1.58	0.55	0.89
04 prep CBP-1	1	166	0.27	1.60	0.20	1.54	0.99	0.79
05 orig CBP-1	2	23	0.05	1.67	0.04	1.53	0.91	0.85
06 prep CBP-1	2	90	0.12	1.60	0.09	1.53	0.99	0.76
07 orig CBP-2a	0	14	6.9	0.60	6.8	0.62	0.11	0.82
08 prep CBP-2a	0	112	18.1	0.69	17.4	0.64	0.51	0.73
09 orig CBP-2a	1	6.9	3.76	0.60	3.75	0.63	0.10	0.66
10 prep CBP-2a	1	75	11.7	0.69	11.2	0.64	0.51	0.73
11 orig CBP-2a	2	3.4	2.05	0.60	2.05	0.60	0.00	0.00
12 prep CBP-2a	2	50	7.61	0.69	7.54	0.56	0.71	0.61
13 orig CBP-2b	0	131	5.20	4.19	4.82	4.14	0.48	0.78
14 prep CBP-2b	0	253	4.26	2.51	3.90	2.46	0.80	0.44
15 orig CBP-2b	1	51	2.15	4.24	2.07	4.06	0.20	0.83
16 prep CBP-2b	1	147	2.37	2.51	2.17	2.46	0.80	0.44
17 orig CBP-2b	2	20	0.92	4.24	0.89	4.06	0.17	0.84
18 prep CBP-2b	2	84	1.31	2.51	1.20	2.46	0.80	0.44
19 orig CBP-3	0	341	154	−2.15	135	−2.15	1.00	0.50
20 prep CBP-3	0	234	97	−2.07	86	−2.01	0.94	0.54
21 orig CBP-3	1	289	23	1.25	17	1.26	0.86	0.57
22 prep CBP-3	1	195	73	−2.07	65	−2.01	0.94	0.54
23 orig CBP-3	2	245	17.4	1.25	13.1	1.26	0.86	0.57
24 prep CBP-3	2	163	56	−2.07	51	−2.02	0.89	0.56

$$C_i^2(\alpha, a, \kappa) = \sum_{j=0}^{m} \frac{N_{\mathrm{max},j}^2}{m-1}$$

$$f(x) = A_0 \exp(-u^2/2) \text{ with}$$

$$u = (x - N_{max}),$$

$$C_{\mathrm{MHS}}(\alpha, a, \kappa) = \mathrm{Min}(C_i^2(\alpha, a, \kappa))$$

$$L_{\mathrm{MHS}}(\alpha, a, \kappa) = C_{\mathrm{MHS}} \cdot I_{\mathrm{uncurled}}^{-n}$$

Optimum parameter set is found by a Simplex Downhill minimization

Summary: Semi-automatic method

- Select a structure in coronal or chromospheric image.
- Choose footpoint areas in aligned magnetogram.
- Compute MHS solution with arbitrary parameters alpha, a, kappa.
- Compute (many) magnetic field lines which connect the selected footpoint areas and compare them (functional L) with coronal loops.
- Use a Simplex downhill method to find minimum of L with respect to alpha, a, kappa.
- We can add to L additional contrains, e.g. additional chromospheric images or vector magnetograms.

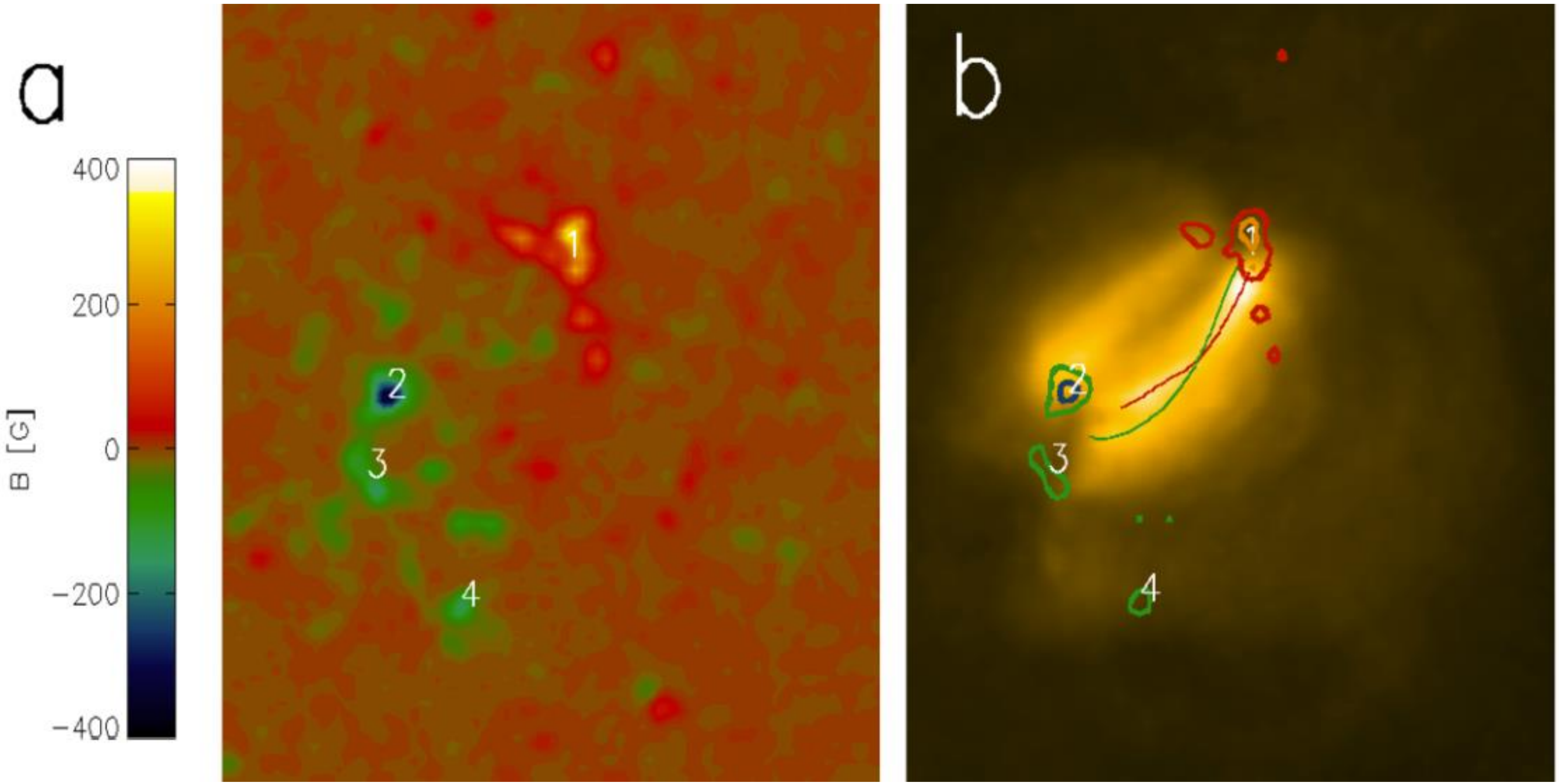
Fully automatic method

- Aim: Use a magnetogram and a coronal image (HMI+AIA) and the algorithm should find the optimum linear MHS solution without any human intervention.
- The code does:

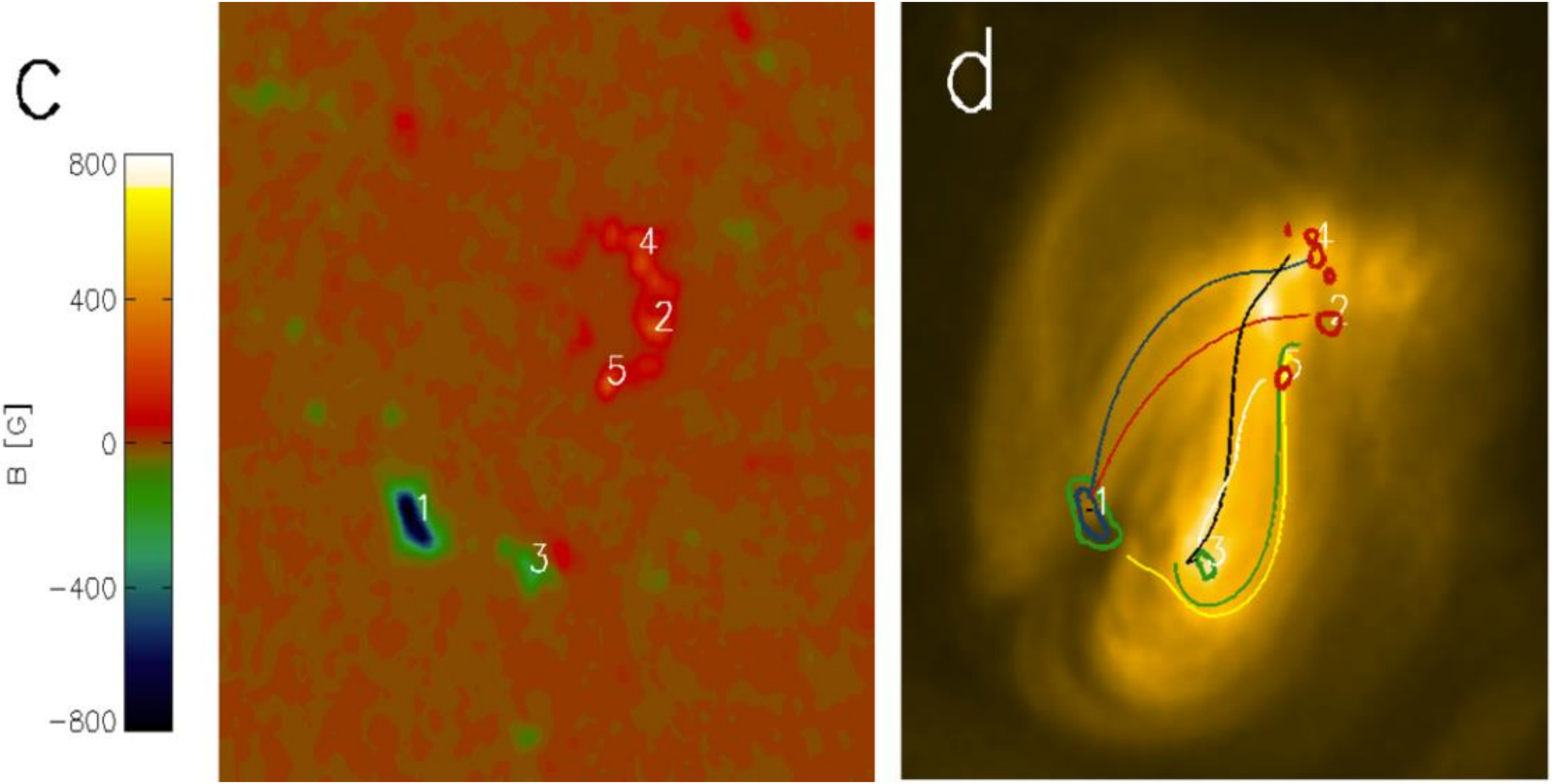
-Identify Magnetic Elements in the Magnetogram.

-Identify Pairs of Magnetically Connected Elements.

-Optimize the Free LMHS Parameters.



a
b
400
200
0
−200
−400
B [G]
1
2
3
4


c
800
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-400
-800
B [G]
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4
5
d
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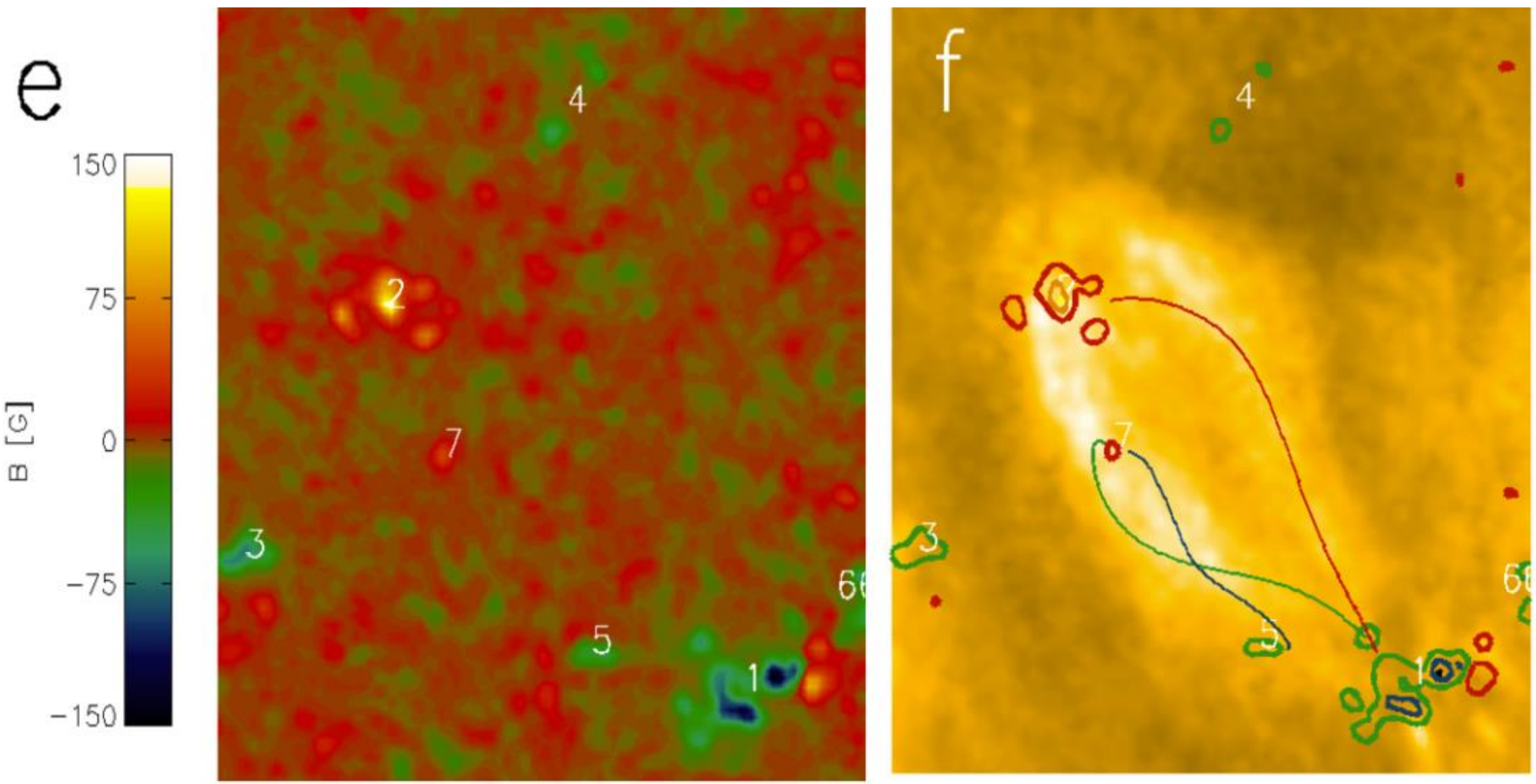
4
5



e
f
150
75
0
−75
−150
B [G]
1
2
3
4
5
6
7

Case	n	L_{Pot}	L_{LFF}	α_{LFF}	L_{MHS}	α	a	κ
CBP-1 (1,2)	1	0.24	0.04	2.35	0.04	2.31	0.51	0.78
CBP-1 (1,3)	1	59.5	1.72	3.28	1.54	3.22	0.74	0.73
CBP-1 (1,4)	1	599	165	4.44	139	5.17	0.40	0.34
CBP-2 (2,1)	1	17.2	11.9	−0.74	13.1	−0.74	0.50	0.65
CBP-2 (2,3)	1	341	27.1	4.33	25.5	4.33	0.77	0.50
CBP-2 (4,1)	1	6.71	6.33	0.04	6.33	0.07	0.00	0.00
CBP-2 (4,3)	1	*******	22.2	4.92	21.4	4.92	0.50	0.50
CBP-2 (5,1)	1	201	66.8	4.22	47.4	4.21	0.98	0.48
CBP-2 (5,3)	1	57.3	0.11	2.38	0.11	2.41	0.00	0.00
CBP-3 (2,1)	1	369	122	−2.50	94.7	−2.23	0.42	0.56
CBP-3 (2,3)	1	552	162	1.94	144	1.86	0.68	0.76
CBP-3 (2,4)	1	292	239	0.92	191	0.84	0.62	0.61
CBP-3 (2,6)	1	*******	166	5.02	183	5.04	0.59	0.54
CBP-3 (7,1)	1	479	24.5	2.12	20.5	2.19	0.52	0.66
CBP-3 (7,3)	1	674	364	−2.79	289	−2.30	0.51	0.50
CBP-3 (7,5)	1	131	76.7	−0.54	82.6	−0.54	1.00	0.50

Conclusions

✓ Special classes of magneto-hydro-static equilibria are useful when vector magnetograms are not available (e.g. quiet Sun).

✓ We developed a method to automatically find footpoint areas magnetic loops and by an uncurling method we can compare the magnetic model and the coronal image.

✓ A Simplex Downhill method finds the optimum set of the 3 free parameters (alpha, a, kappa).

✓ If the image shows several loops, the optimum parameter set is usually different for each structure.