Solar MHS modeling with optimization method and MHD relaxation method

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Low's MHS solution

- **MHS equations:** $\begin{cases} \boldsymbol{j} \times \boldsymbol{B} - \nabla p + \rho \boldsymbol{g} = 0 \\ \boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{\mu_0 j} \\ \nabla \cdot \boldsymbol{B} = 0 \end{cases}$

- **Low's assumption (Low 1991):** $\boldsymbol{\nabla} \times \boldsymbol{B} = f(z)\boldsymbol{\nabla} B_z \times \boldsymbol{e_z} + \alpha_0 \boldsymbol{B}$
  - First component: Electric current perpendicular to gravitational force
  - Second component: Electric current parallel to the magnetic field
  - $f(z) = ae^{-\kappa z}$ controls the intensity and characteristic length of Lorentz force
  - Free parameters: α_0, a, κ

- **Solution:**

$$\begin{cases} \tilde{B}_x = \dfrac{1}{h^2 + k^2}\left(ih\dfrac{d\tilde{B}_z}{dz} + ik\alpha_0 \tilde{B}_z\right) \\ \tilde{B}_y = \dfrac{1}{(h^2 + k^2)}\left(ik\dfrac{d\tilde{B}_z}{dz} - ih\alpha_0 \tilde{B}_z\right) \\ p = p_0(z) - \dfrac{1}{8\pi} f(z) B_z^2 , \\ \rho = -\dfrac{1}{g}\dfrac{dp_0}{dz} + \dfrac{1}{4\pi g}\left[\dfrac{1}{2}\dfrac{df}{dz} B_z^2 + f(\boldsymbol{B} \cdot \boldsymbol{\nabla}) B_z\right] \end{cases}$$

Reference model 1

Our first reference model is built with the following inputs:

- LOS magnetogram at the lower boundary: In principle, any observed LOS magnetogram can be used as the lower boundary, but to make the first reference model smooth and relatively easy to compute, we extract the LOS magnetogram from Low and Lou (1990)'s nonlinear force-free field (with parameters $n = 1$, $m = 1$, $l = 0.3$, and $\Phi = 0.47$) on $z = 0$ plane bounded by $x, y \in [-1, 1]$.
- Parameters for the MHS model: $\alpha = -0.3$, $a = 0.5$, and $\kappa = 0.02$.
- Background atmosphere: Ideal gas with $\rho_0 = 2.7 \times 10^{-7} g/cm^3$ on the photosphere and the temperature $T = 6000K/5500K/10000K$ at height $h = 0Mm/0.5Mm/2Mm$ (linear interpolation at points between the nodes). The mean molecular weight $M = 1$ and the gravitational acceleration $g = 272.2 m/s^{-2}$ over the active region.
- Computational box: $-1.6Mm < x, y < 1.6Mm$ and $0Mm < z < 2Mm$ resolved with $80 \times 80 \times 50$ grid points.

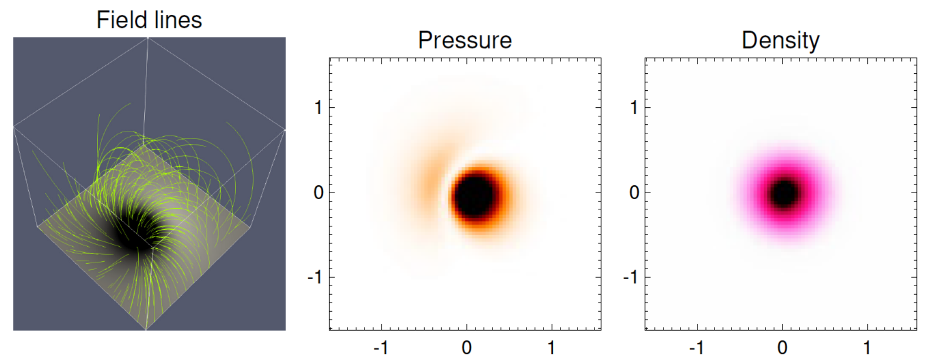

Figure 1: Low's analytical model. Left: Magnetic field lines with the LOS magnetogram at the bottom. Middle: LOS integration of plasma pressure. Right: LOS integration of plasma density.

- In this test case the plasma density the vector magnetogram for all six boundaries are specified for extrapolation. Plus, the pressure scale height in the 3D volume is also specified for extrapolation to close the MHS equations.

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The optimization method for the MHS extrapolation

- NLFFF equations: $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} = 0$

  $\nabla \cdot \boldsymbol{B} = 0$
- Functional: $L = \int_V [B^2\Omega_a^2 + |\nabla \cdot \boldsymbol{B}|^2)]dV$

  where: $\boldsymbol{\Omega_a} = \frac{[(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}]}{B^2}$
- Boundary condition: vector magnetogram
- Initial condition: potential field
- $minimize\ L(\boldsymbol{B})$

- MHS equation: $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \nabla p - \rho\hat{z} = 0$

  $\nabla \cdot \boldsymbol{B} = 0$
- Functional: $L = \int_V [B^2\Omega_a^2 + |\nabla \cdot \boldsymbol{B}|^2)]dV$

  where: $\boldsymbol{\Omega_a} = \frac{[(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \nabla p - \rho\hat{z}]}{B^2 + p}$
- Boundary condition: vector magnetogram + plasma distribution $(p + \frac{B_z^2}{2} = p_{quiet})$
- Initial condition: NLFFF + atmosphere (trace the plasma distribution along the magnetic field lines)
- $minimize\ L(\boldsymbol{B}, p, \rho)$

$p = Q^2, \qquad \rho = R^2$

$minimize\ L(B, Q, R)$

Changes in algorithm for the test

- Since scale height **h** at all grid points are specified in this test:

$$h = \frac{1}{g}\frac{RT}{M} = \frac{1}{g}\frac{p}{\rho}.$$

➡

- MHS equation: $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \nabla p - (p/h)\hat{z} = 0$

$$\nabla \cdot \boldsymbol{B} = 0$$

- Functional: $L = \int_V [B^2 \Omega_a^2 + |\nabla \cdot \boldsymbol{B}|^2)] dV$

where: $\boldsymbol{\Omega_a} = \frac{[(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \nabla p - (p/h)\hat{z}]}{B^2 + p}$

- $minimize\ L(\boldsymbol{B}, Q)\ where\ p = Q^2$

Numerical implementation:

1. **Calculate a potential field with Bn on all 6 boundaries**
2. **Calculate a NLFFF by using optimization method with the vector magnetogram on all 6 boundaries**
3. **Create a gravity stratified atmosphere**
4. **Iterate $(\vec{\boldsymbol{B}}, \boldsymbol{Q})$ until L reaches minimum**

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The MHD relaxation method for the MHS extrapolation

- An evolution method that by slowly changing the bottom magnetic field to drive the MHD system to evolve, and finally converges to a stationary state

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot \left[\rho \boldsymbol{v}\boldsymbol{v} + \left(p + \frac{\boldsymbol{B}\boldsymbol{B}}{2}\right)\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}\right] = \rho \boldsymbol{g} - \mu \rho \boldsymbol{v},$$

$$\frac{\partial E}{\partial t} + \nabla \cdot \left[\left(E + p + \frac{\boldsymbol{B} \cdot \boldsymbol{B}}{2}\right)\boldsymbol{v} - \boldsymbol{B}(\boldsymbol{B} \cdot \boldsymbol{v})\right] = -\mu \rho v^2,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = 0,$$

- ☐ Numerical scheme:
  - ✓ Total Variation Diminishing Lax-Friedrichs (TVDLF)
  - ✓ 2^{nd} order accuracy in time and space
- ☐ Remove the $\nabla \cdot B$ by solving Poisson equation with conjugate gradient method
- ☐ Boundary condition:
  - ✓ Bottom: “Stress and Relax”
  - ✓ Others: potential field
- ☐ Friction coefficient: $\mu = 0.1$

Changes in algorithm for the test

- Since scale height **h** at all grid points are specified in this test:

$$h = \frac{1}{g}\frac{RT}{M} = \frac{1}{g}\frac{p}{\rho}.$$

$\Rightarrow$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot \left[\rho \boldsymbol{v}\boldsymbol{v} + \left(p + \frac{\boldsymbol{B}\boldsymbol{B}}{2}\right)\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}\right] = \rho \boldsymbol{g} - \mu \rho \boldsymbol{v},$$

$$p = \rho g h$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = 0,$$

Numerical implementation:

1. **Calculate a potential field with Bn on all 6 boundaries**
2. **Create a gravity stratified atmosphere**
3. **Slowly change the magnetic field on all 6 boundaries to drive the system, until it converges**

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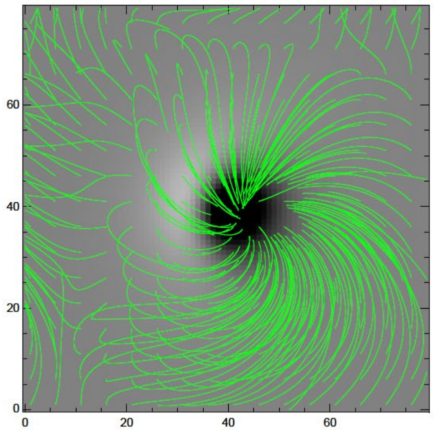
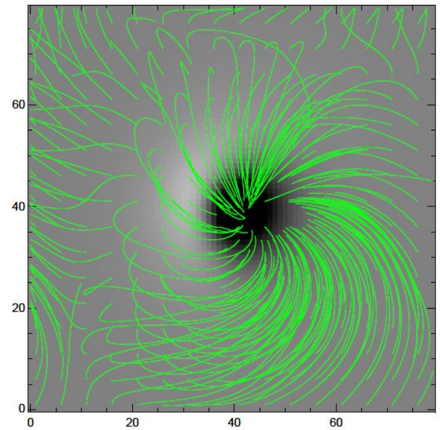
Results: Magnetic field

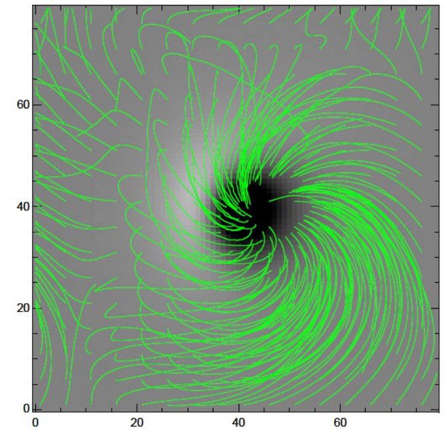
Reference Model



A

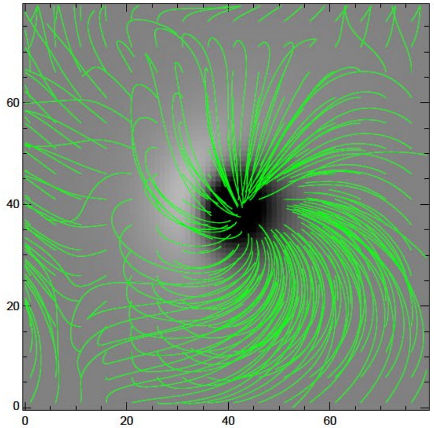


B

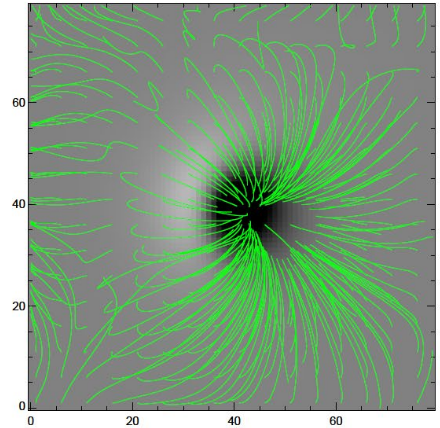


- ☐ **MHS by MHD**
- ☐ **MHS by opt.**
- ☐ **NLFFF by opt.**
- ☐ **Potential field**

C



D

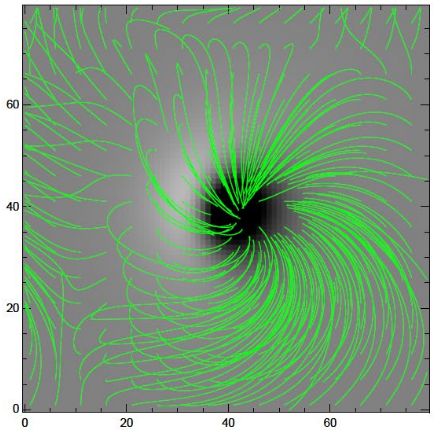


Potential field: D

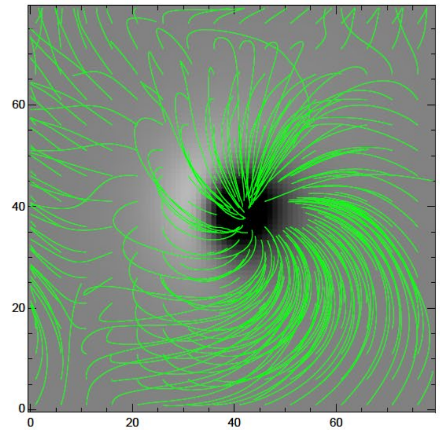
B is over-twisted?

Results: Magnetic field

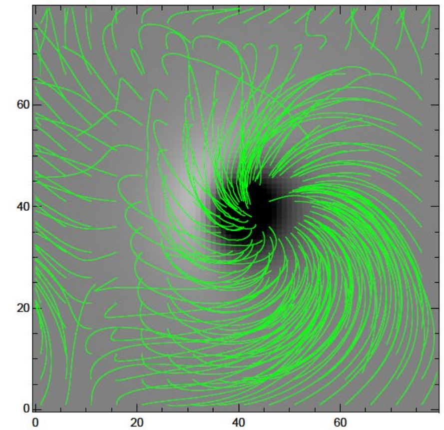
Reference Model



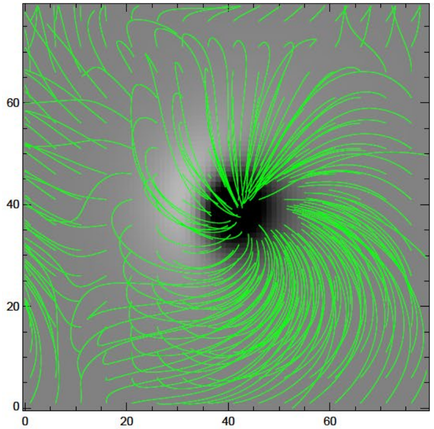
MHS by optimization



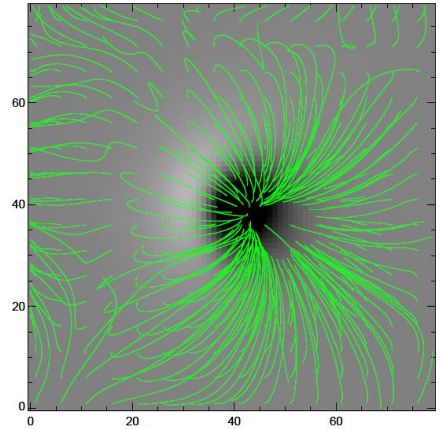
NLFFF by optimization



MHS by MHD relaxation



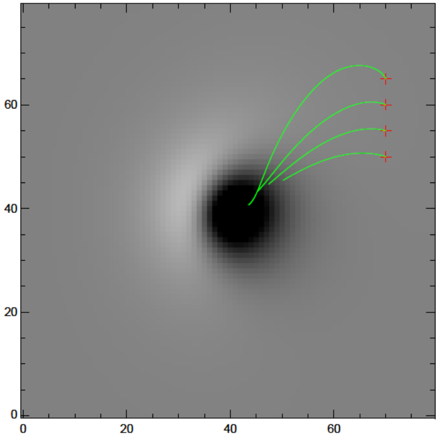
Potential field



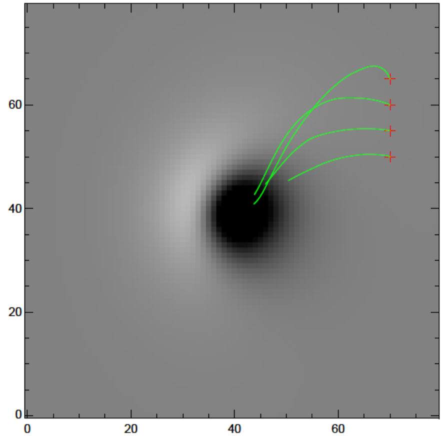
- Potential field lines lack twist
- NLFFF looks a bit over-twisted
- MHS extrapolations perform better than NLFFF

Results: Magnetic field

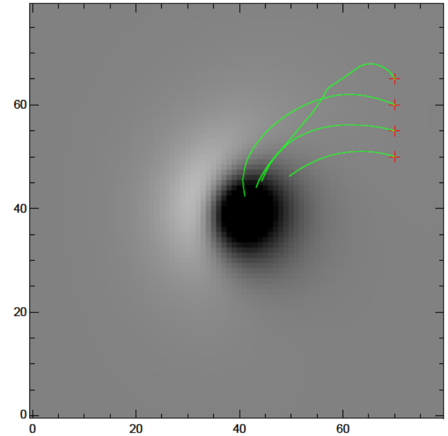
Reference Model



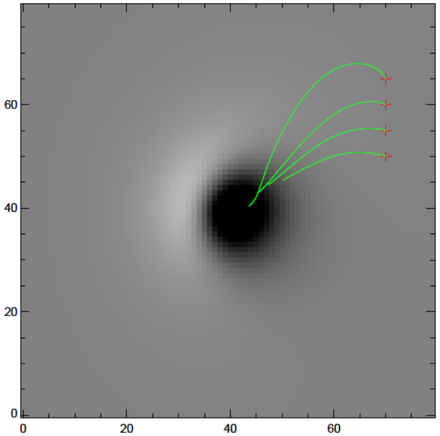
MHS by optimization



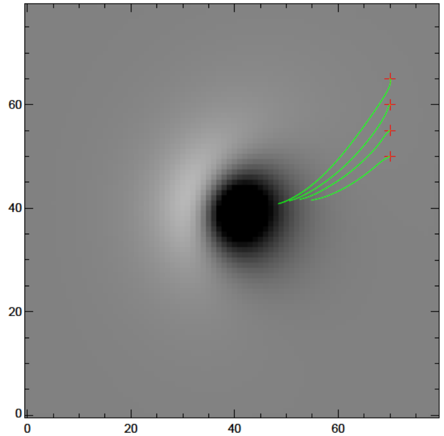
NLFFF by optimization



MHS by MHD relaxation



Potential field

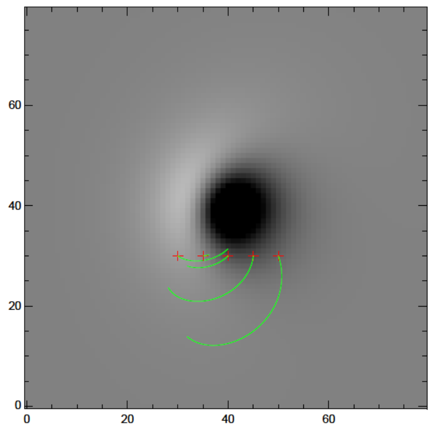


Visual rank:

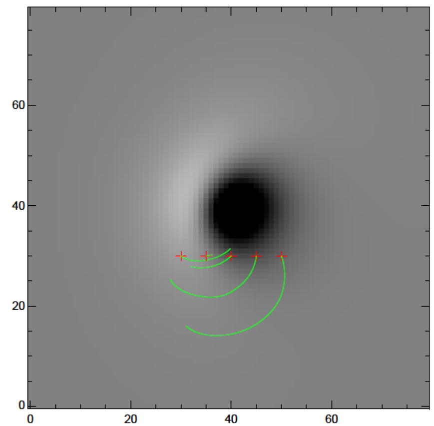
1. MHS by MHD relaxation
2. MHS by optimization
3. NLFFF by optimization
4. Potential field

Results: Magnetic field

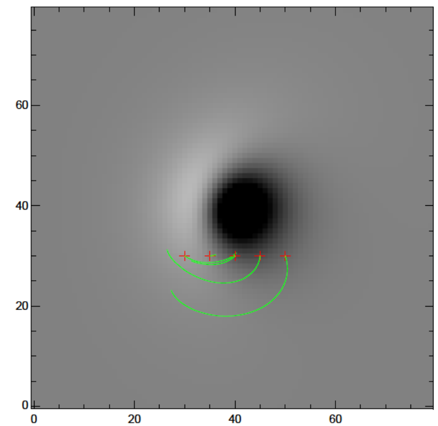
Reference Model



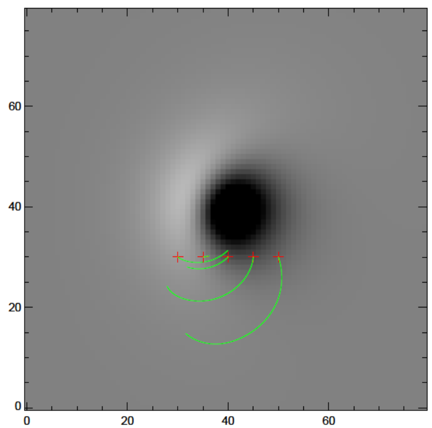
MHS by optimization



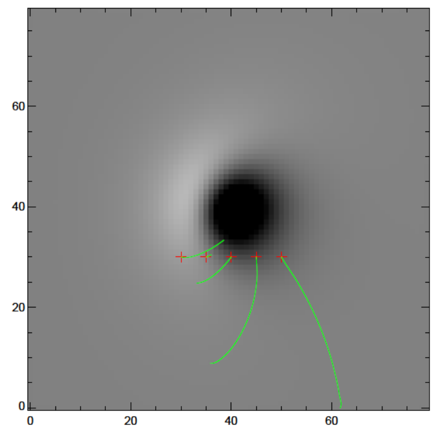
NLFFF by optimization



MHS by MHD relaxation



Potential field

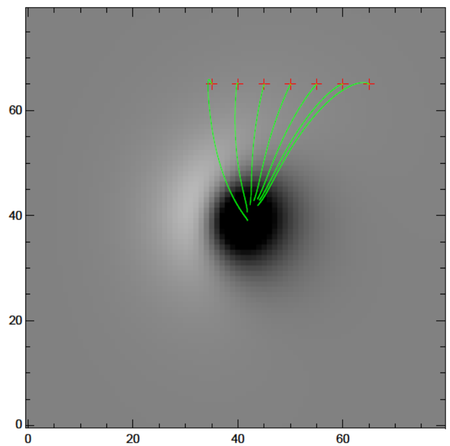


Visual rank:

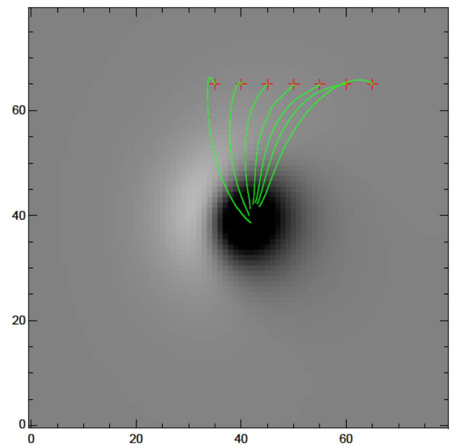
1. Two MHS methods
2. NLFFF by optimization
3. Potential field

Results: Magnetic field

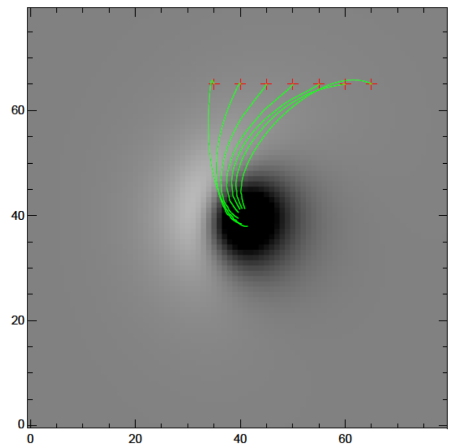
Reference Model



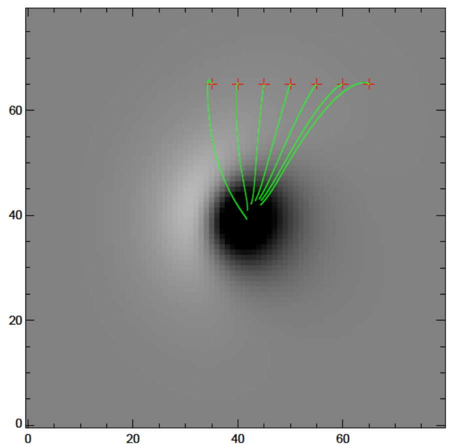
MHS by optimization



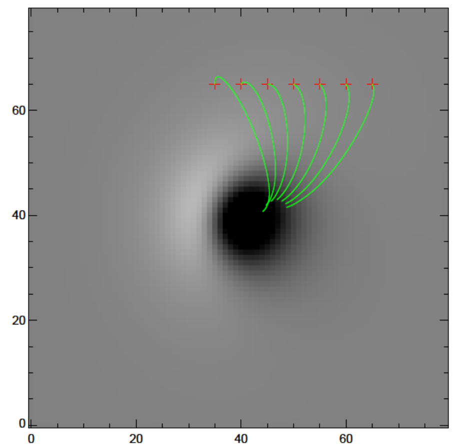
NLFFF by optimization



MHS by MHD relaxation



Potential field



Visual rank:

1. MHS by MHD relaxation
2. MHS by optimization
3. NLFFF by optimization
4. Potential field

Results: Magnetic field

$$C_{\mathrm{vec}} \equiv \sum_i \boldsymbol{B}_i \cdot \boldsymbol{b}_i \Big/ \left(\sum_i |\boldsymbol{B}_i|^2 \sum_i |\boldsymbol{b}_i|^2 \right)^{1/2}.$$

$$C_{\mathrm{CS}} \equiv \frac{1}{M} \sum_i \frac{\boldsymbol{B}_i \cdot \boldsymbol{b}_i}{|\boldsymbol{B}_i||\boldsymbol{b}_i|} \equiv \frac{1}{M} \sum_i \cos\theta_i$$

$$|f_i| = |(\nabla \cdot \boldsymbol{B})_i| / (6|\boldsymbol{B}|_i/\Delta x),$$

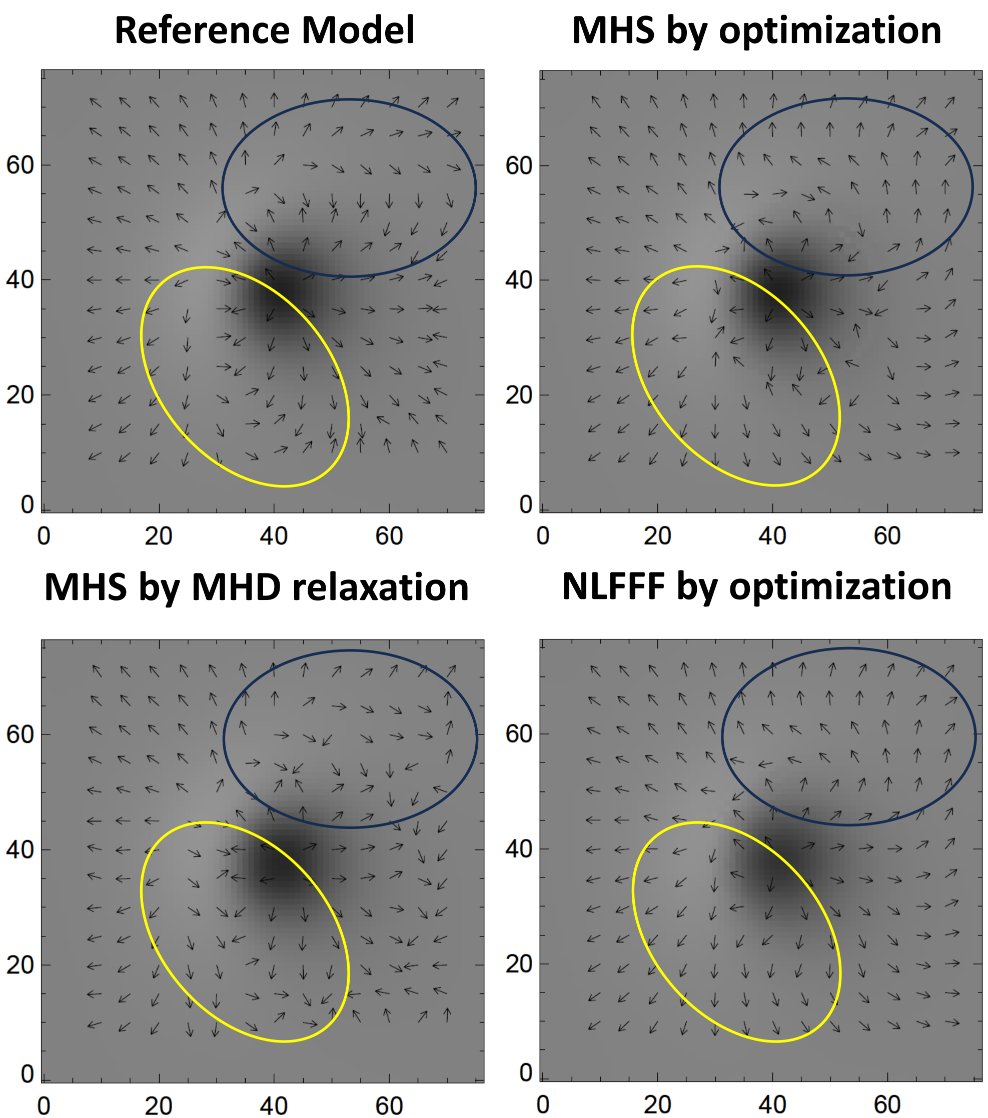
$$E_{\mathrm{n}} = \sum_i |\boldsymbol{b}_i - \boldsymbol{B}_i| \Big/ \sum_i |\boldsymbol{B}_i|$$

$$E_{\mathrm{m}} = \frac{1}{M} \sum_i \frac{|\boldsymbol{b}_i - \boldsymbol{B}_i|}{|\boldsymbol{B}_i|}.$$

- E/Er: Magnetic energy of extrapolation over magnetic energy of reference model
- FLD: Field Line Divergence, a score to evaluate how well the topology is recovered

Model	C_{vec}	C_{CS}	1-En	1-Em	E/Er	<fi>$\times 10^4$	1-FLD(A/F)
Ref. Field	1	1	1	1	1	32.8	1/1
Pot. Field	0.94	0.82	0.56	0.46	0.78	14.8	0.21/0.13
Nlff by Optim.	0.98	0.97	0.81	0.82	0.81	24.7	0.71/0.44
MHS by Optim.	1.00	0.99	0.92	0.90	1.03	32.8	0.94/0.90
MHS by MHD	1.00	1.00	0.95	0.94	0.99	31.8	0.96/0.96

Results: Lorentz force



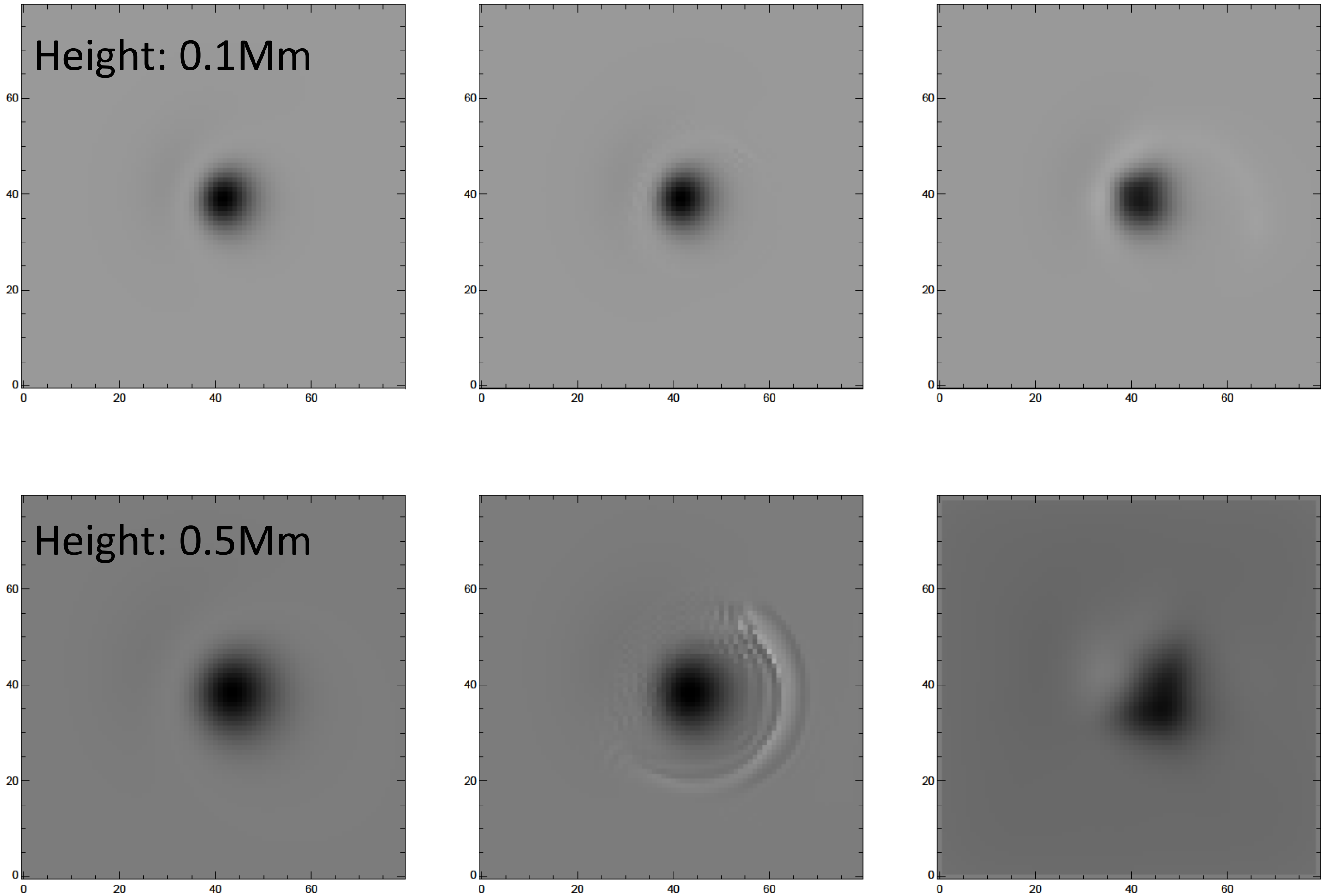
Lorentz force vector at height 0.2 Mm

Metric calculated in the whole region

Model	C_{vec}	C_{CS}	1-En
Ref. Field	1	1	1
Nlff by Optim.	0.97	0.46	0.40
MHS by Optim.	0.99	0.64	0.74
MHS by MHD	0.99	0.69	0.78

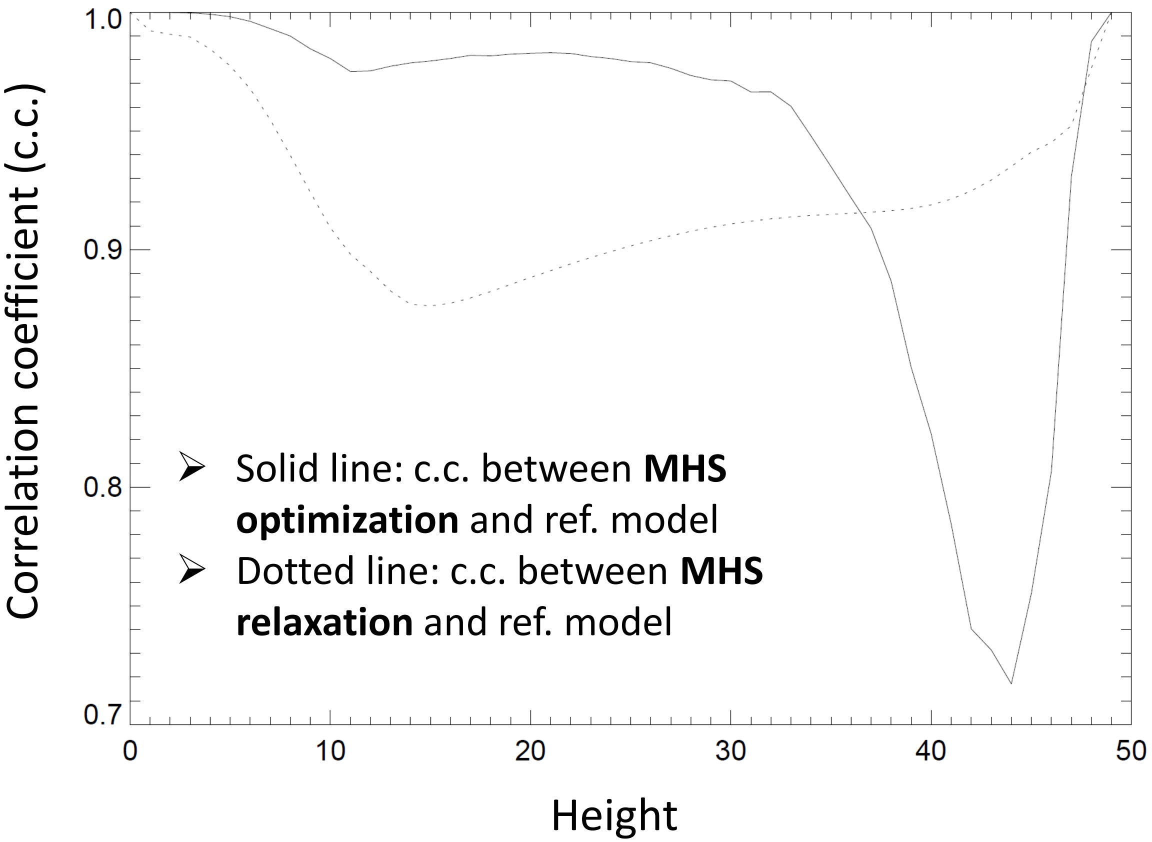
Results: Plasma pressure

Reference Model | **MHS by optimization** | **MHS by MHD relaxation**



- Plasma pressure close to the bottom boundary well recovered
- Optimization: Circular depletion area well recovered, but with ring oscillation
- MHD relaxation: No oscillation, but the circular depletion area is not well recovered

Results: Plasma pressure



- The optimization method performs better overall, but the performance decreases quickly over height

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Summary

- In the Low's test case, the MHS extrapolations reconstruct accurate magnetic field (better than the NLFFF).
- The performance in recovering the Lorentz force is also acceptable.
- The plasma pressure can be recovered accurately near the bottom boundary. The result is not very good as the height increase.
- Keep in mind that we use all of boundary conditions, plus temperature condition in 3D box as input.

THE END