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Solar & Magnetospheric Theory Group
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Analytical 3D MHS Equilibria: Theoretical Background

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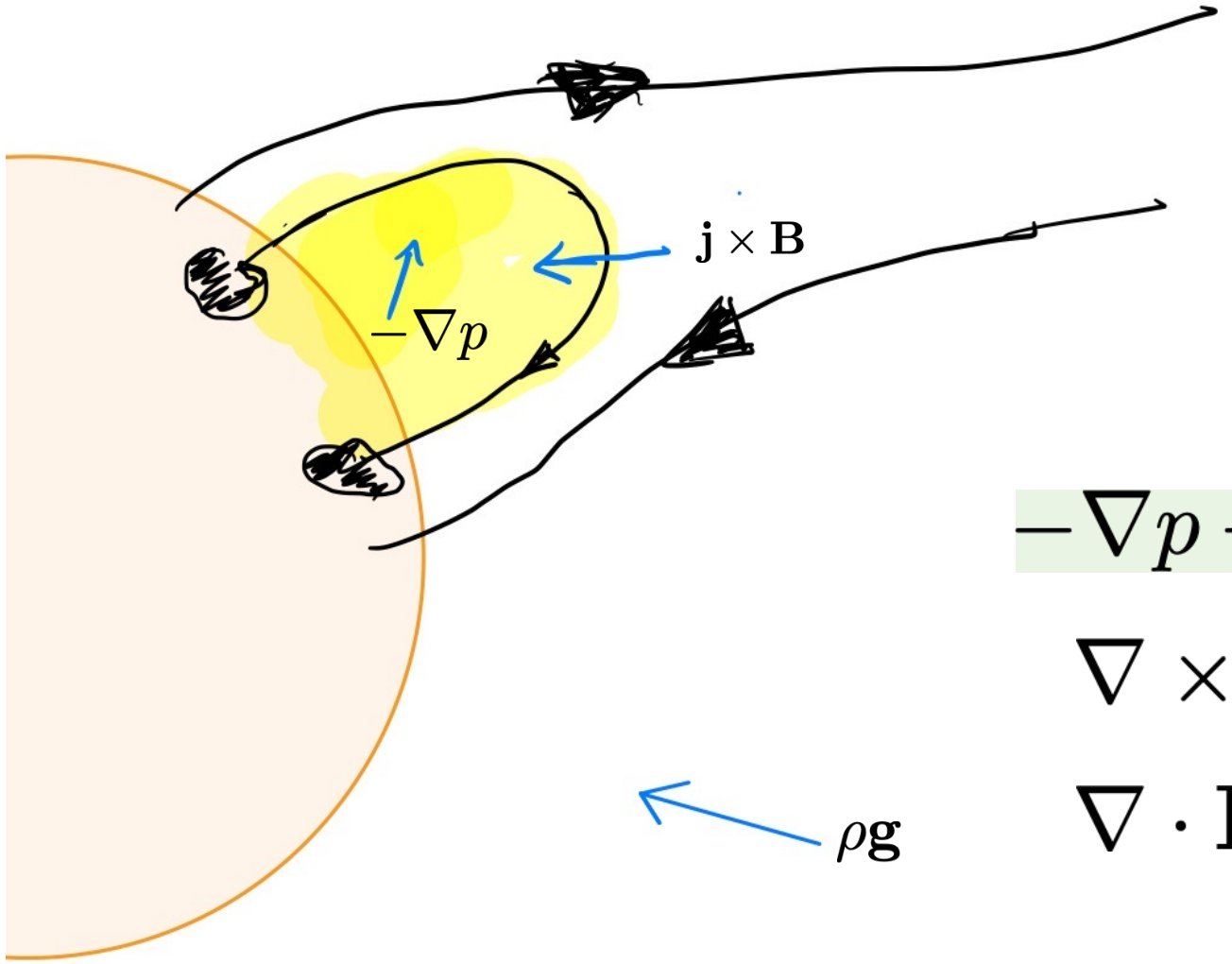
Force Balance



$$-\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g} = \mathbf{0}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

$$\nabla \cdot \mathbf{B} = 0$$

Magnetohydrostatic (MHS) Methods

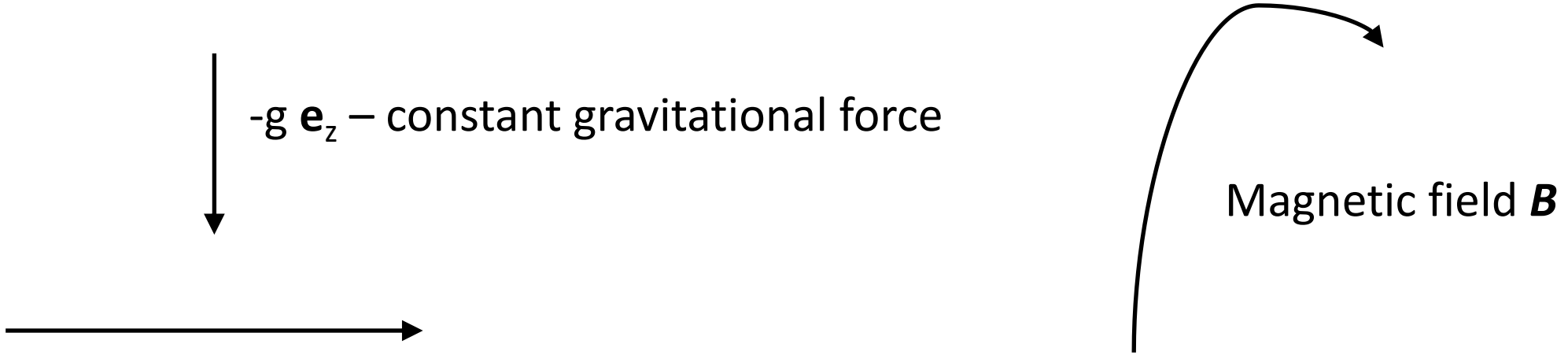
- Fully nonlinear numerical codes for magnetohydrostatic (MHS) extrapolation

  (e.g. Wiegelmann & TN, 2006; Gilchrist & Wheatland, 2013; Gilchrist et al., 2016; Zhu et al., 2013; Zhu & Wiegelmann, 2018, 2022; Mathews et al., 2022)

- Any (analytical) “short-cuts” (“rough’n’ready”, but sufficient for “quick-look”)?

Analytical 3D MHS equilibria (Cartesian case)

- Following work by B.C. Low and others:

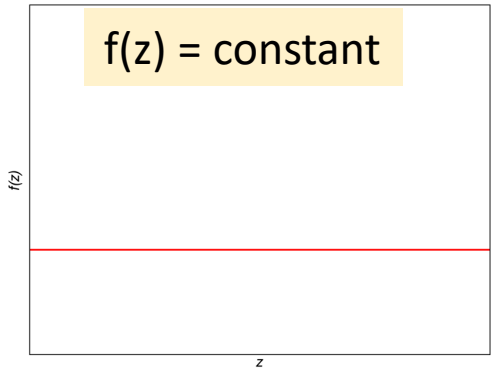


Current density $\boldsymbol{j} =$ f(z) $\nabla B_z \times \boldsymbol{e}_z$ + $\alpha\, \boldsymbol{B}$

⊥ to direction of gravitational force || to ***B***

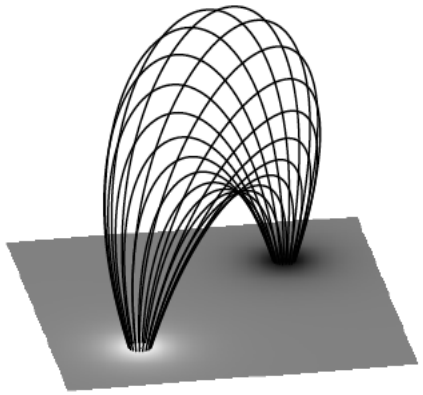
Free function f(z) controls perpendicular current density in this model !

Two Examples

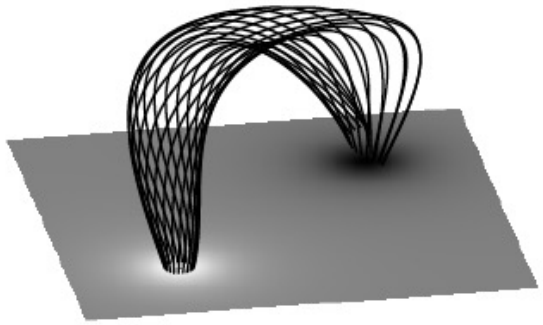


z-dependence of B:

exponential functions

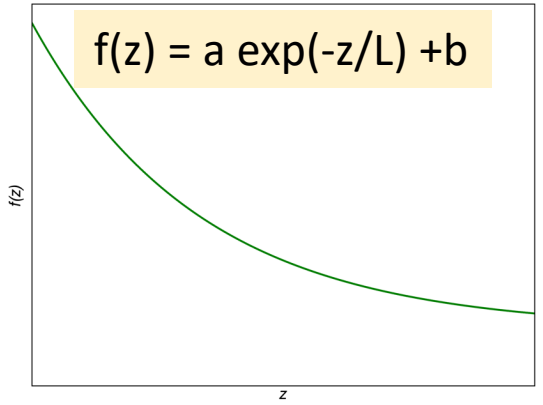


potential



MHS

Petrie & TN, 2000
Petrie, PhD thesis, 2000

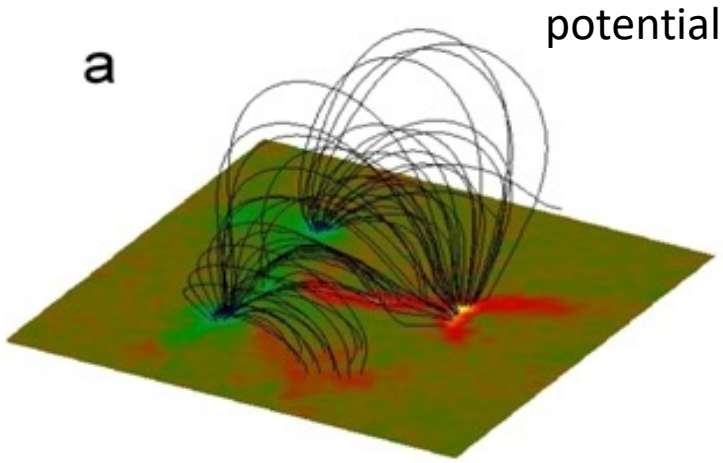


z-dependence of B:

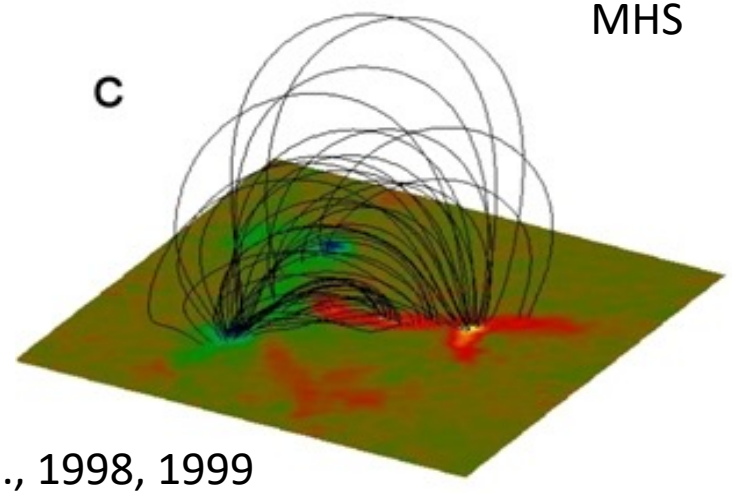
Bessel functions
(e.g. Low 1991)

Wiegelmann et al., 2015

potential



MHS



See also e.g. Aulanier et al., 1998, 1999

Can we “control” *j* even more?

(details see TN & Wiegelmann, 2019)

Parameters:

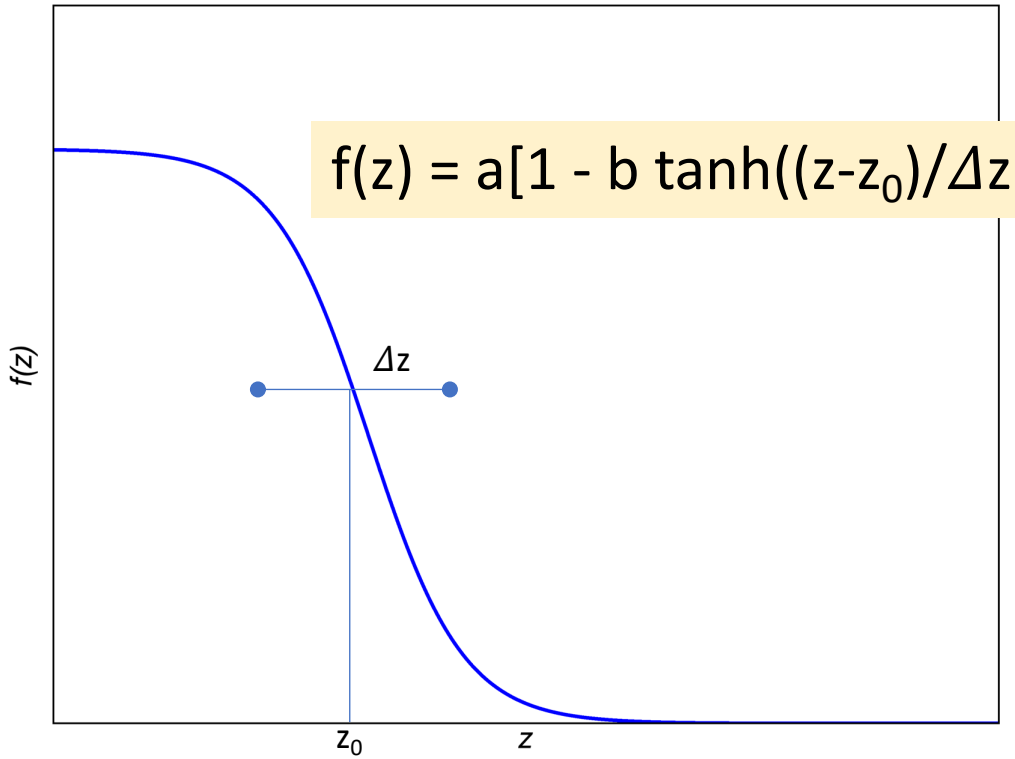
a: “amplitude” parameter

b: “switch-off” parameter

z_0: height of “switch off”

Δz: width of ”switch off”

(Remark: total “switch off” only for b = 1.0)



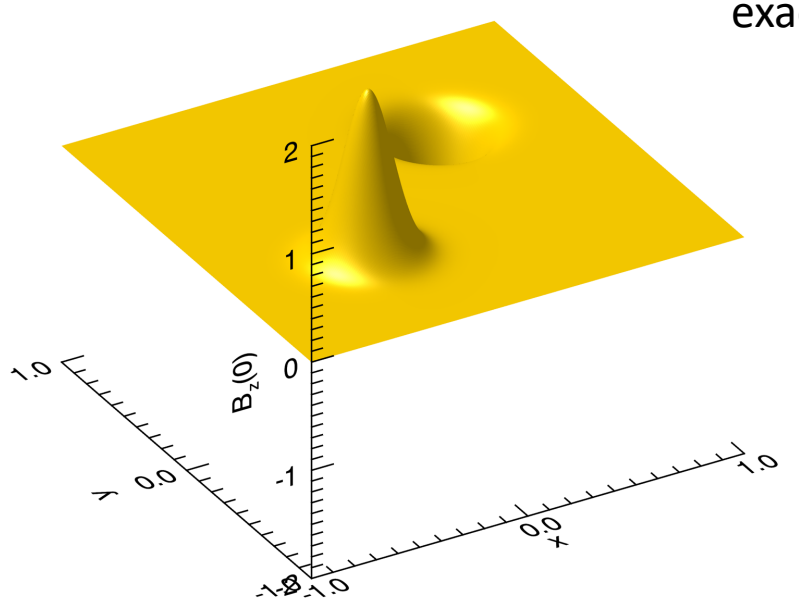
z-dependence of B :

hypergeometric functions

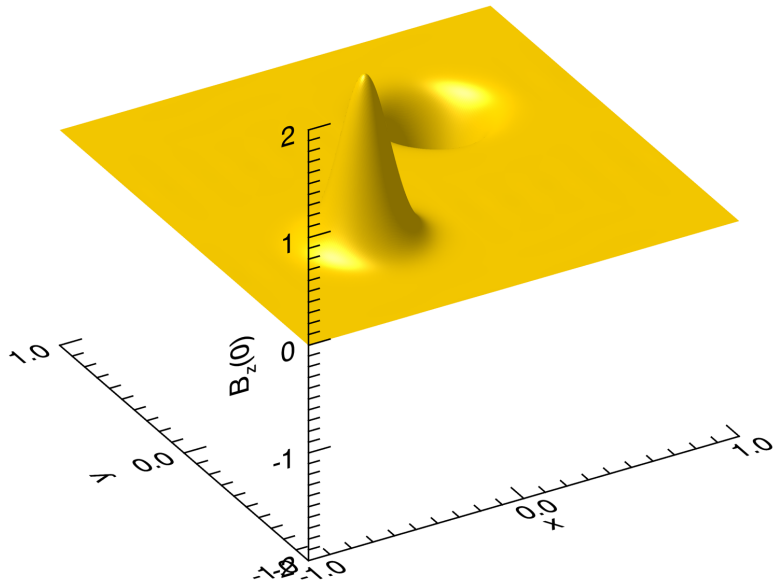
“Toy magnetogram” (periodic “Gaussian”)

Periodic in x and y, easily expanded into Fourier modes (based on von Mises distribution)

exact



10 Fourier modes

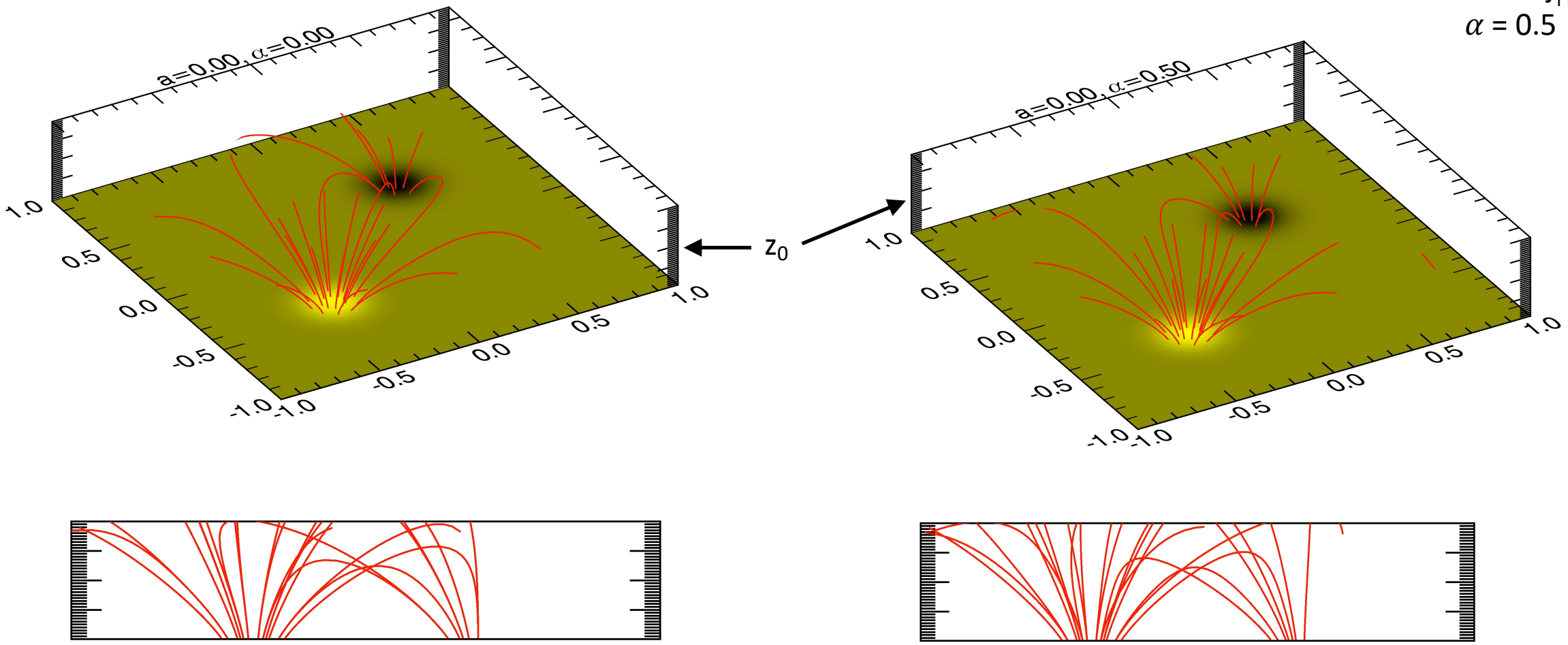


$$B_z(x,y,0) \propto e^{[\kappa_x \cos(x+\mu_x)]} e^{[\kappa_y \cos(y+\mu_y)]} - e^{[\kappa_x \cos(x-\mu_x)]} e^{[\kappa_y \cos(y-\mu_y)]}$$

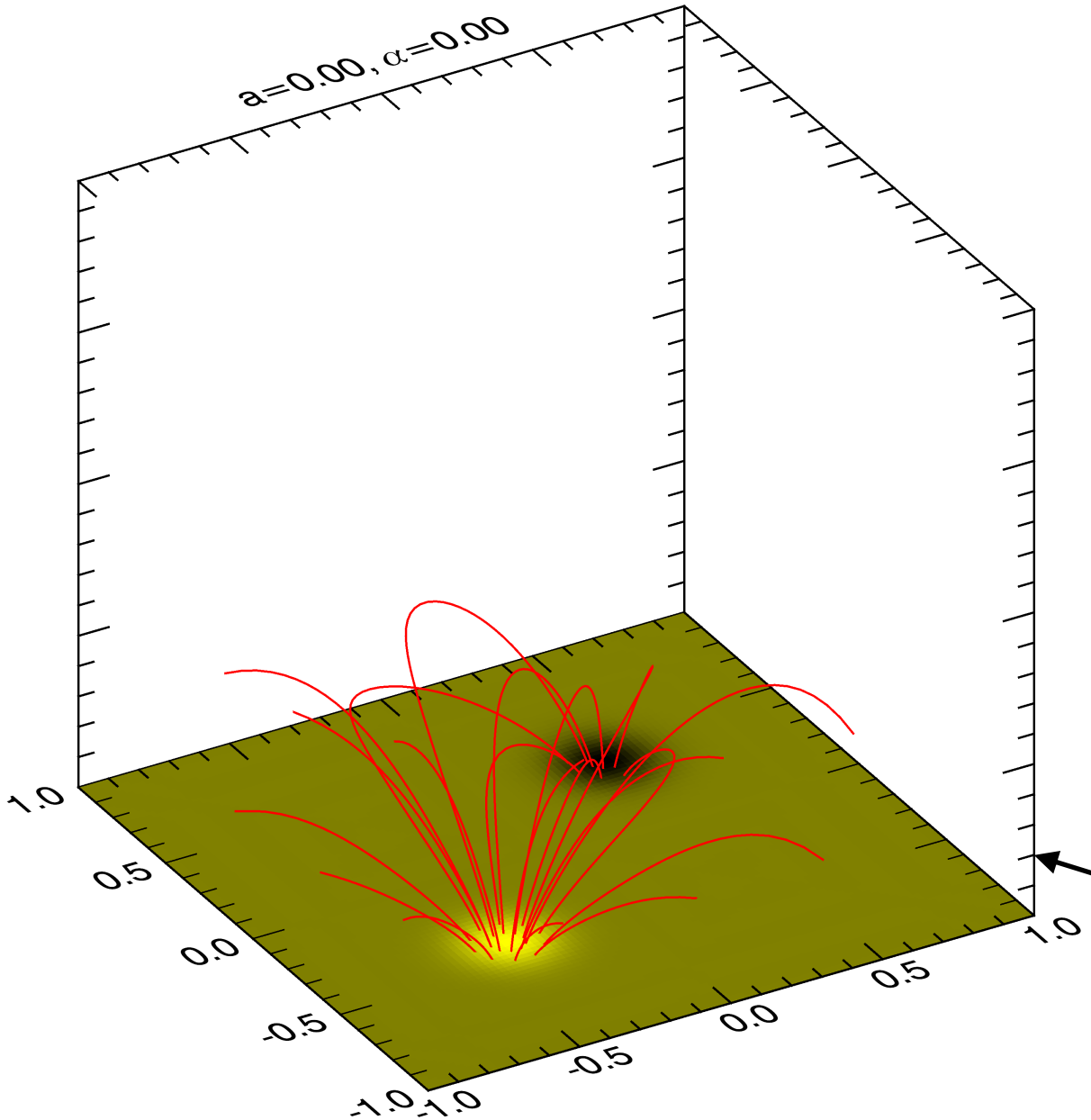
- Parameter a increases from 0.0 to 0.49 (upper bound is 0.5)
- b = 1.0 (potential field for z >> z_0)
- α = 0.0 (left), α = 0.5 (right)
- z_0 = 0.2
- $\Delta z = 0.1\ z_0$

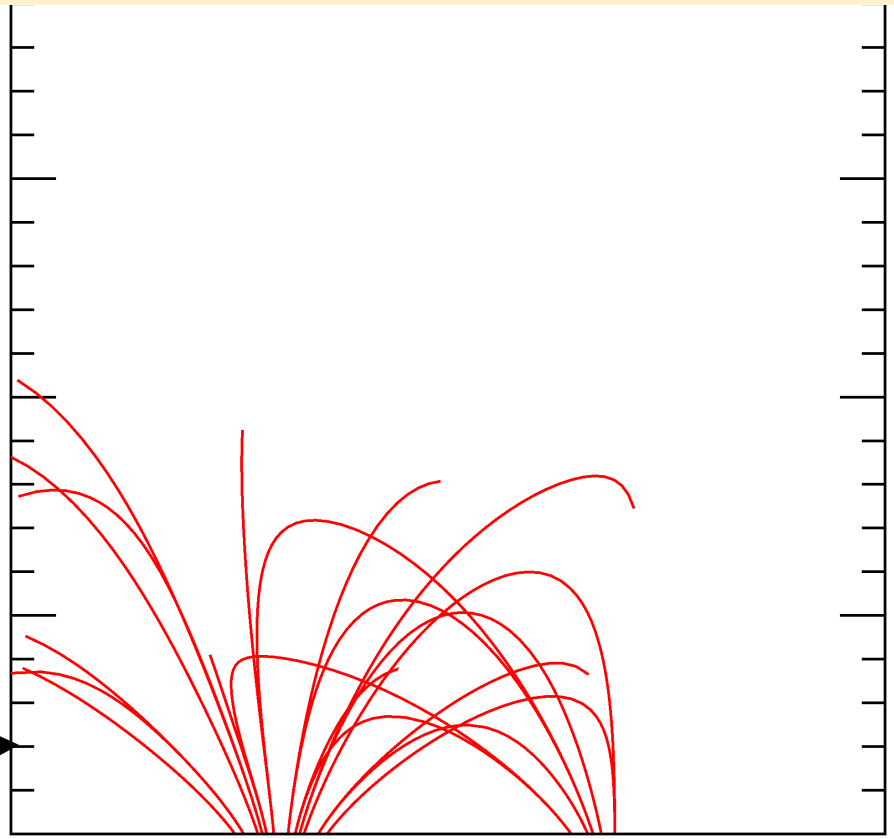
No $j_{||}$, α = 0

Finite $j_{||}$, α = 0.5



- Parameter a increases from 0.0 to 0.49 (upper bound is 0.5)
- b = 1.0 (potential field for $z >> z_0$)
- α = 0.0
- z_0 = 0.2
- $\Delta z = 0.1\ z_0$





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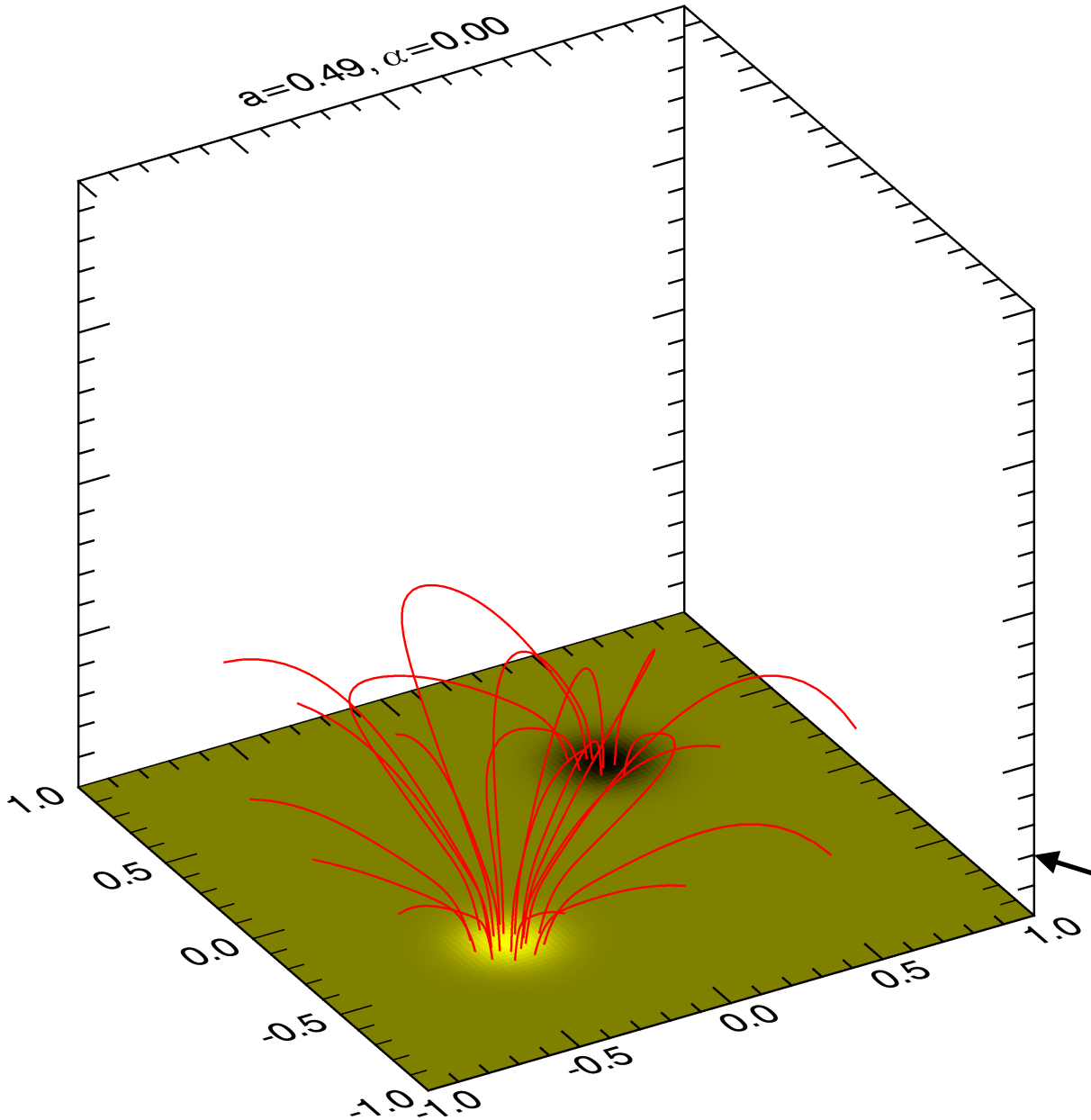
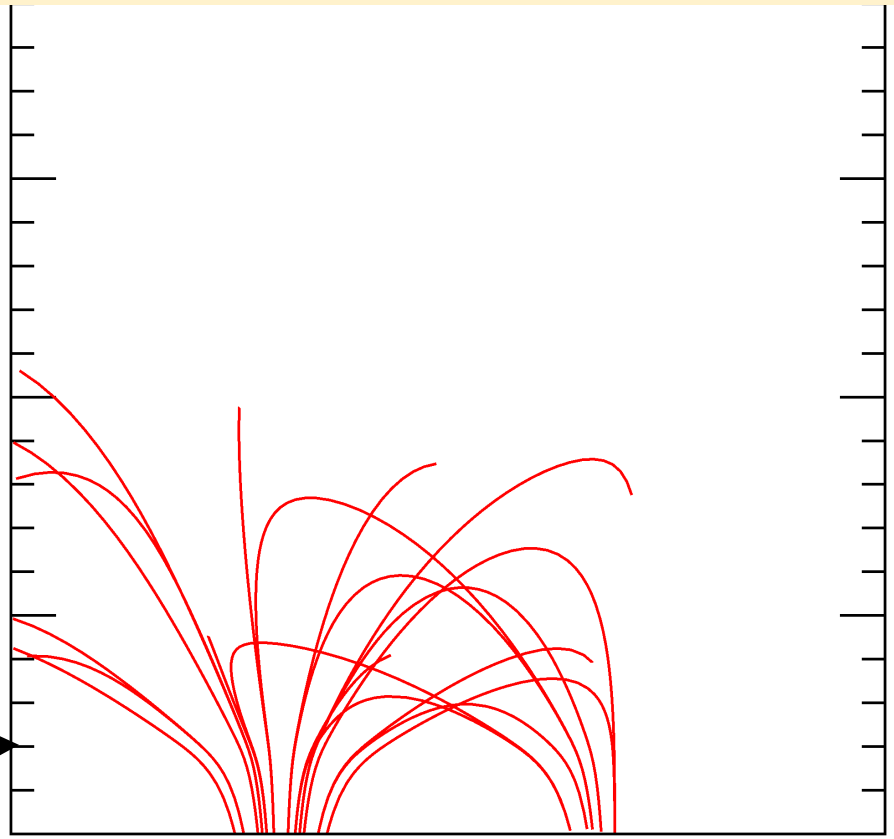
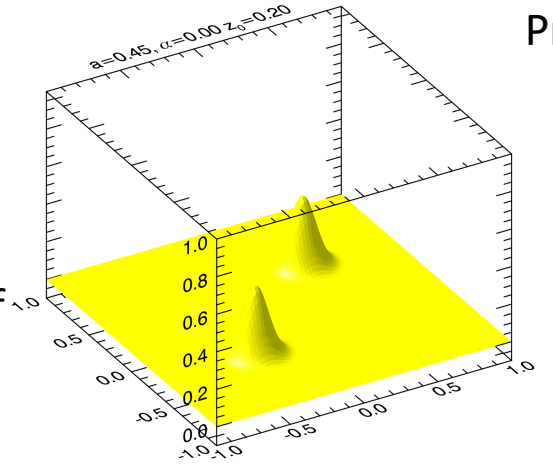




Pressure and Density

z_0=0.2

z=0.1
below
“switch-off

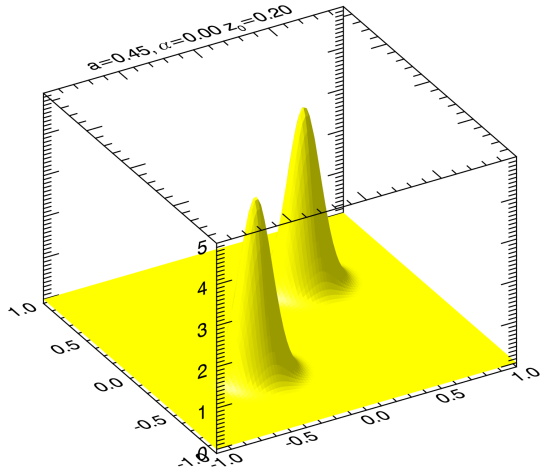


Pressure variation

$$p = p_0(z) + \Delta p$$

$$\Delta p = -f(z)\frac{B_z^2}{2\mu_0}$$

Shown: $-\Delta p$



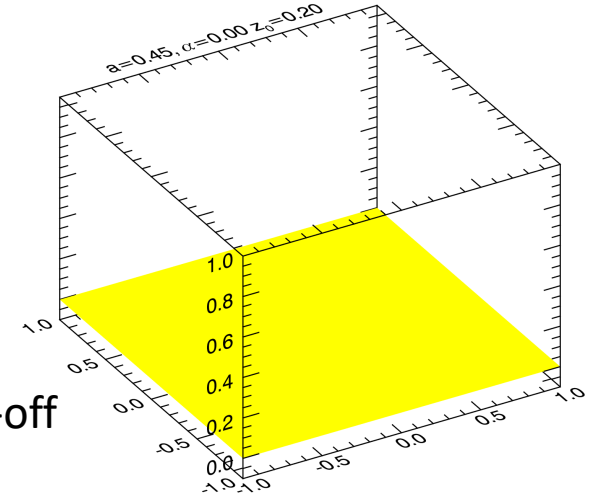
Density variation

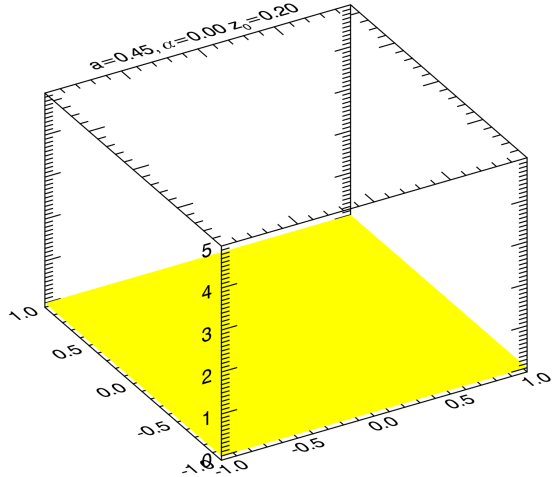
$$\rho = \rho_0(z) + \Delta\rho$$

$$\Delta\rho = \frac{\mathrm{d}f}{\mathrm{d}z}\frac{B_z^2}{2\mu_0} + \frac{f}{\mu_0}\mathbf{B}\cdot\nabla B_z$$

Shown: $-\Delta\rho$

z=0.4
above
“switch-off





Application of method to other astrophysical systems

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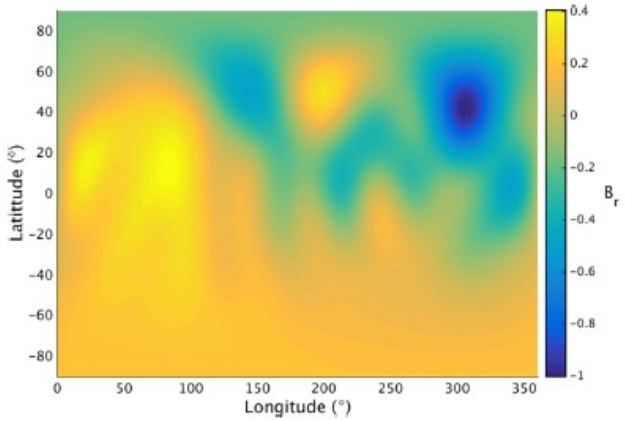
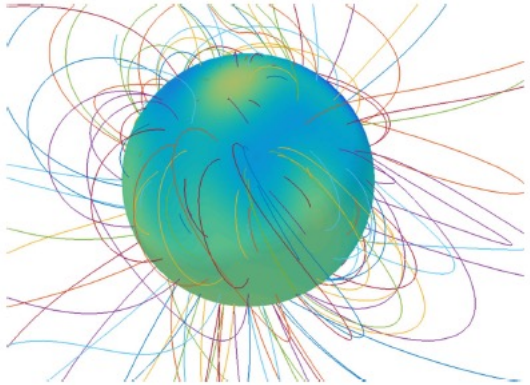
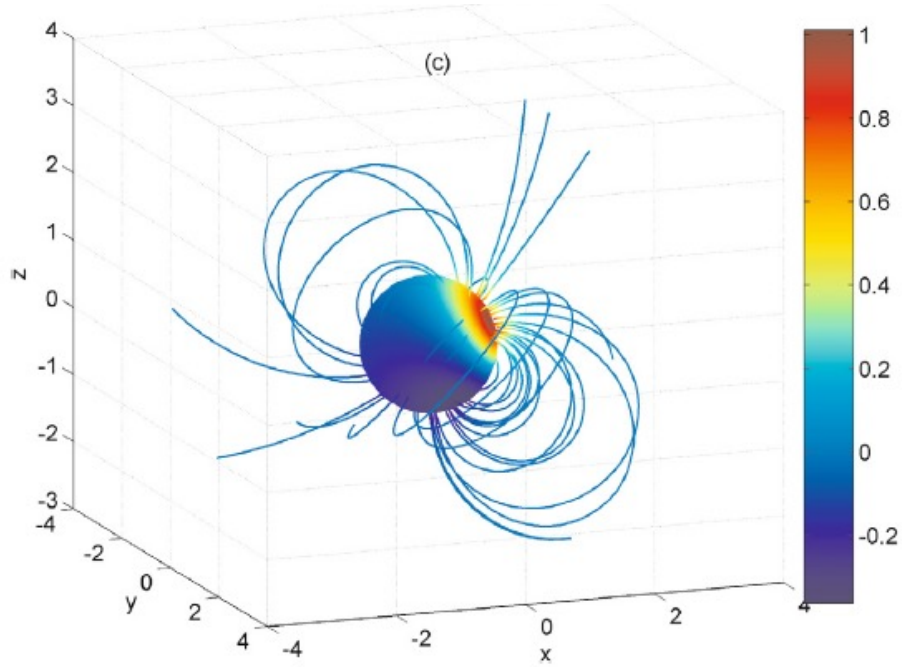




Without rotation

MacTaggart et al. (2016)

Artificial magnetic surface field model:
Displaced and tilted magnetic dipole



With rotation

Al-Salti & TN (2010)

Numerical !

Summary

- Showed a few examples of the use of analytical 3D MHS equilibria in solar physics
- Numerical methods essential due to the nonlinear nature of the problem, e.g. for magnetic field extrapolation
- For MHS, there is a possibility for using analytical methods to complement numerical methods
- Same approach could be useful for other applications, e.g. stellar magnetic field models

Thank you for listening!

Happy to answer any questions