

Validation of the magnetohydrostatic extrapolation methods with known models

ISSI/ISSI-BJ Team: Magnetohydrostatic modeling of the solar
atmosphere with new datasets

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1 The Magnetohydrostatic Equations

The magnetohydrostatic (MHS) equations to be solved can be written as:

$$\mathbf{J} \times \mathbf{B} - \nabla p - \rho \mathbf{g} = 0, \quad (1)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (3)$$

where ρ is the mass density, p is the plasma pressure, \mathbf{B} is the magnetic field, \mathbf{J} is the electric current density, and $g = 272.2m/s^2$ is the gravitational acceleration. The above system of equations are underdetermined given that it has more variables than equations. To close the system, we might relate pressure and density with temperature (T) via an equation of state

$$p = \rho \frac{RT}{M}, \quad (4)$$

where R is the ideal gas constant and M is the mean atomic weight. In principle, we need an energy-balance equation to treat the unknown T by dealing with plasma heating and cooling owing to radiation, thermal conduction, viscous dissipation, resistive dissipation, etc. Some of the above physics processes are still unclear to us. Thus, in practice, the MHS modeling ignores the energy-balance equation and prescribes a pressure scale height distribution (Gilchrist et al., 2016; Miyoshi et al., 2020) or leaves the system underdetermined (Zhu et al., 2013; Zhu and Wiegmann, 2018). The formula of the pressure scale height is:

$$h = \frac{1}{g} \frac{RT}{M} = \frac{1}{g} \frac{p}{\rho}. \quad (5)$$

A well-set boundary conditions as discussed in Gilchrist and Wheatland (2013) for the MHS equation are:

$$\mathbf{B} \cdot \hat{\mathbf{n}}|_{\partial V} = B_n, \quad (6)$$

$$\mathbf{J} \cdot \hat{\mathbf{n}}|_{\pm\partial V} = J_n, \quad (7)$$

$$p|_{\pm\partial V} = p_{bnd}, \quad (8)$$

where $\pm\partial V$ is either $+\partial V$ where $B_n > 0$ or $-\partial V$ where $B_n < 0$. This means normal component of \mathbf{B} is prescribed over ∂V , while p and normal component of \mathbf{J} are prescribed at only one polarity.

2 Reference models

2.1 Reference model 1: Low's semi-analytical MHS model

Low (1991) describes a special class of 3D solutions for the MHS equations (1)-(3). The idea is to prescribe special electric currents to reduce the MHS equations to the following tractable form:

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} + ae^{-\kappa z} \nabla B_z \times \hat{\mathbf{z}}, \quad (9)$$

where α , a , and κ are constants in space. The above equation has two current systems: linear force-free current and current being everywhere perpendicular to the gravitational force. We can obtain a full 3D solution having prescribed magnetic field (B_z) obtained from magnetograms, α , a and κ .

Our first reference model is built with the following inputs:

- LOS magnetogram at the lower boundary: In principle, any observed LOS magnetogram can be used as the lower boundary, but to make the first reference model smooth and relatively easy to compute, we extract the LOS magnetogram from Low and Lou (1990)'s nonlinear force-free field (with parameters $n = 1$, $m = 1$, $l = 0.3$, and $\Phi = 0.47$) on $z = 0$ plane bounded by $x, y \in [-1, 1]$.
- Parameters for the MHS model: $\alpha = -0.3$, $a = 0.5$, and $\kappa = 0.02$.
- Background atmosphere: Ideal gas with $\rho_0 = 2.7 \times 10^{-7} g/cm^3$ on the photosphere and the temperature $T = 6000K/5500K/10000K$ at height $h = 0Mm/0.5Mm/2Mm$ (linear interpolation at points between the nodes). The mean molecular weight $M = 1$ and the gravitational acceleration $g = 272.2m/s^{-2}$ over the active region.
- Computational box: $-1.6Mm < x, y < 1.6Mm$ and $0Mm < z < 2Mm$ resolved with $80 \times 80 \times 50$ grid points.

The magnetic fields and plasma distributions are shown in figure (1). In this test case the physical quantities ρ, \mathbf{B} for all six boundaries are specified. Moreover, the pressure scale height (see equation (5)) in the 3D volume is also specified to close the MHS equations.

Comparison between the reference model and the extrapolation results will be made in the entire volume. The MHD relaxation code implemented respectively by Zhu and by Miyoshi,

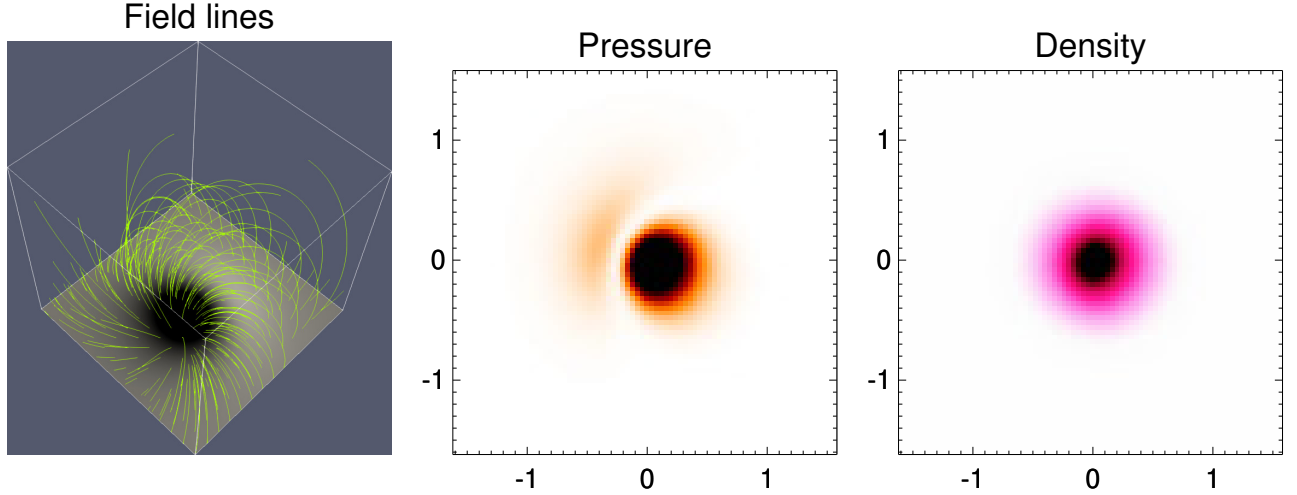


Figure 1: Low’s analytical model. Left: Magnetic field lines with the LOS magnetogram at the bottom. Middle: LOS integration of plasma pressure. Right: LOS integration of plasma density.

the Grad-Rubin code implemented by Wheatland, the optimization code implemented by Zhu and Wiegmann will be applied to the model.

2.2 Reference model 2: Cheung’s RMHD simulation

Cheung et al. (2019) carried out an radiative magnetohydrodynamic (RMHD) simulation of a solar flare along with a coronal mass ejection. The simulation includes 3D radiative transfer in the convection zone and photosphere, along with optically thin radiation and field-aligned heat conduction in the corona, which mimics a realistic environment on the Sun.

We chose a snapshot that is eight minutes after flare peak as another reference model for the test. The photospheric magnetogram is extracted at the same geometrical height at which the average optical depth is equal to unity. Thus the reference model spans ± 49.2 (± 24.6) Mm in x (y) axis and spans from photosphere (0 Mm) to 41.6 Mm above in z axis. Although the system is still somewhat dynamic at the moment of the reference snapshot, inertial term is relatively weak at any height compared with either plasma pressure or magnetic pressure (see figure 2 in Zhu and Wiegmann, 2022). The reference model is resolved by $512 \times 256 \times 652$ grid points with grid spacing 192 km in the horizontal directions and 64 km in the vertical.

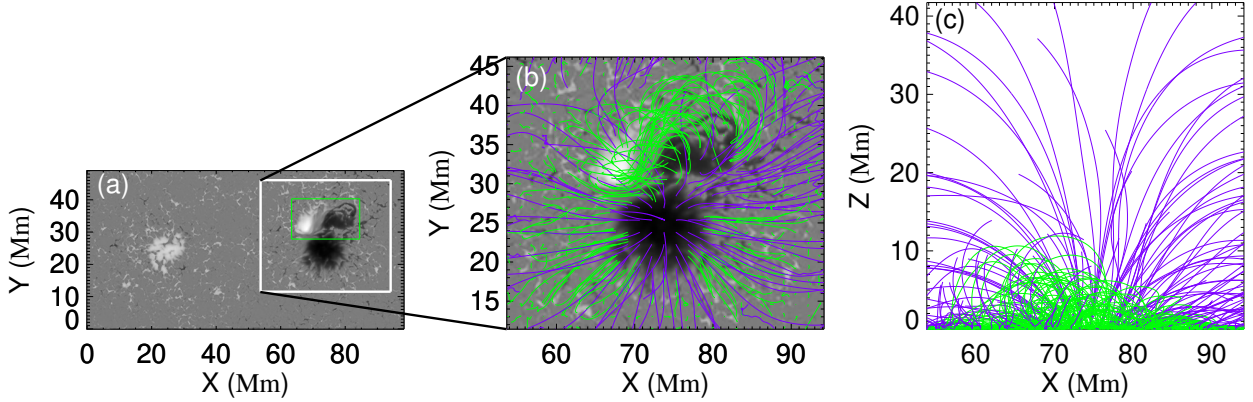


Figure 2: Cheung's RMHD model.

The magnetic field lines are shown in figure (2). In this test case only the vector magnetic field \mathbf{B} in the photosphere is specified. As mentioned in the first section, the MHS equations (1) - (3) with the bottom vector magnetogram is not a well posed problem. Therefore, we have to make more assumptions, not only on the boundaries but also in the computational domain. Since each of our methods has its own way to deal with technical problems, I will not, of course, impose more restrictions here. Please try your best to implement your algorithms with reasonable assumptions to get the best results. If you are going to use an atmosphere model as the initial condition or to constrain your initial condition, I would suggest the background atmosphere of the 1st reference model with an extension to corona: $T = 500,000K/1,000,000K/1,500,000K$ at height $h = 2.2Mm/5Mm/100Mm$ (linear interpolation at points between the nodes).

Comparison between the reference model and the extrapolation results will be made in the emerging spot within $[280:490,60:240,1:156]$ (green box in figure 2). The Linear MHS solution established by Neukirch, the MHD relaxation code implemented respectively by Zhu and by Miyoshi, the Grad-Rubin code implemented by Wheatland, and the optimization code implemented by Zhu and Wiegmann will be applied to the reference model 2 for comparison.

3 Metrics for Comparison

The following metrics which were frequently used for quantitative comparison between the reference model and extrapolation results will be used in our comparison:

- Vector Correlation metric

$$C_{vec} = \sum_i \mathbf{B}_i \cdot \mathbf{b}_i / \left(\sum_i |\mathbf{B}_i|^2 \sum_i |\mathbf{b}_i|^2 \right)^{\frac{1}{2}}, \quad (10)$$

- Cauchy-Schwarz metric

$$C_{CS} = \frac{1}{N} \sum_i \frac{\mathbf{B}_i \cdot \mathbf{b}_i}{|\mathbf{B}_i| |\mathbf{b}_i|}, \quad (11)$$

- Normalized Vector Error metric

$$E_N = \sum_i |\mathbf{B}_i - \mathbf{b}_i| / \sum_i |\mathbf{B}_i|, \quad (12)$$

- Mean Vector Error metric

$$E_M = \frac{1}{N} \sum_i \frac{|\mathbf{B}_i - \mathbf{b}_i|}{|\mathbf{B}_i|}, \quad (13)$$

- Magnetic Energy metric

$$\epsilon = \frac{b^2}{B^2}, \quad (14)$$

where \mathbf{b} and \mathbf{B} are the magnetic field of the reconstruction solution and the reference model, respectively, N is the number of grid points in the computation box.

- Field Line Divergence (FLD) metric: Tracing field lines from a random point on the bottom boundary in both reference model and extrapolation. If both field lines end again on the bottom boundary, a score p_i can be given to this point with the distance between the two endpoints divided by the length of the field line in the reference model. Then a single score can be assigned by the fraction of the area in which p_i s are less than 10%. An alternative score can be given by the fraction of flux.
- Linear Pearson correlation coefficient for the line-of-sight integration (with respect to the z axis) of both pressure and density.

4 Tasks for each member

We will have our first meeting in the middle of 2023. The main topic of the first meeting is to discuss results of each model. Tasks for each member before the meeting are in the bold fonts in the following table.

Name	Task
Bai, Xianyong	<ul style="list-style-type: none"> • DKIST observation application • Magnetic field inversion
Chifu, Iulia	<ul style="list-style-type: none"> • DKIST observation application • 3D coronal loop reconstruction • NLFFF modelling
Gafeira, Ricardo	<ul style="list-style-type: none"> • Neural network • Magnetic field inversion
Miyoshi, Takahiro	<ul style="list-style-type: none"> • MHS modeling by MHD relaxation method for reference model 1 and 2
Neukirch, Thomas	<ul style="list-style-type: none"> • Linear MHS modeling for reference model 2
Wheatland, Mike	<ul style="list-style-type: none"> • MHS modeling by Grad-Rubin method for reference model 1 and 2
Wiegelmann, Thomas	<ul style="list-style-type: none"> • NLFFF, LMHS, and MHS modeling
Zhao, Jie	<ul style="list-style-type: none"> • COSPAR-2024 event proposition • Magnetic field modeling of the solar active region
Zhu, Xiaoshuai	<ul style="list-style-type: none"> • DKIST observation application • COSPAR-2024 event proposition • MHS modeling by both optimization and MHD relaxation method for reference model 1 and 2

We might need additional runs of our codes after discussions during the first meeting. Our first paper will be on the comparison of the the MHS solutions and the reference solutions.

5 About the data

You may download data named *MHS_test_data.tar.gz* for tests from here. Extract it with command:

```
tar -xzf MHS_test_data.tar.gz
```

After that, you will acquire four .sav files with two **3D_data.sav* are 3D reference models and the other two **boundary_data.sav* contain all known conditions for modelings.

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