Force-free and magnetostatic modelling of NOAA 12723

MAX PLANCK INSTITUTE
FOR SOLAR SYSTEM RESEARCH

Thomas Wiegelmann

- Force-free modelling.
- Magnetostatic modelling.

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Force-free magnetic fields

Magneto-hydro-static equations

$$\mathbf{j} \times \mathbf{B} - \nabla P - \rho \nabla \Psi = \mathbf{0},$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j},$$
$$\nabla \cdot \mathbf{B} = 0.$$

In the coronal low beta plasma we can neglect in lowest order non-magnetic forces like pressure gradients and gravity and derive the (usually nonlinear) **force-free field equations**:

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = \mathbf{0},$$
$$\nabla \cdot \mathbf{B} = 0.$$

NLFFF Code

$$L = \int_V \left[B^{-2}\,|(\nabla \times \mathbf{B}) \times \mathbf{B}|^2 + |\nabla \cdot \mathbf{B}|^2\right] d^3V + \nu \int_S w(x,y)(\mathbf{B} - \mathbf{B}_{\mathbf{obs}})^2 d^2S$$

force | div B | free parameter | B_T error matrix | boundary data

- Compute Potential field in simulation box
- Minimize L numerically
- Bottom boundary B_T becomes injected during iteration
- Injection speed controlled by ν
- Data on boundaries change during iteration
- L=0 corresponds to force-freeness, div B=0 and perfect agreement with boundary data
- For inconsistent data L remains finite, but with a small value of ν we still get an almost force and divergence free configuration

Consistent boundary conditions for force-free fields

(Molodensky 1969, Aly 1989)

$$\int_V \nabla \cdot \mathbf{B}\, d^3x = 0 \Rightarrow \oint_S \mathbf{B}\, d\mathbf{S} = 0$$

Flux-balance,
differential flux-balance

$$\int_V (\nabla \times \mathbf{B}) \times \mathbf{B}\, d^3x = 0$$

$$\int_V \nabla \cdot T\, d^3x = 0 \Rightarrow \oint_S T d\mathbf{S} = 0$$

$$T_{ij} = B_i B_j - \frac{1}{2}\mathbf{B}^2 \delta_{ij}$$

Maxwell Stress Tensor

No net force
on boundary

$$\int_V \mathrm{r} \times [(\nabla \times \mathbf{B}) \times \mathbf{B}]\; d^3x = 0$$

$$\int_V \nabla \cdot \tilde{T}\, d^3x = 0 \Rightarrow \oint_S \tilde{T} d\mathbf{S} = 0$$

$$\tilde{T}_{ij} = \epsilon_{jkl}\, r_k\, T_{ij}$$

No net torque
on boundary

Dimensionless Numbers

1. The flux balance parameter

$$\epsilon_{\text{flux}} = \frac{\int_S B_z \, dx \, dy}{\int_S |B_z| \, dx \, dy}$$

2. The force balance parameter $\epsilon_{\text{force}} =$

$$\frac{|\int_S B_x B_z \, dx \, dy| + |\int_S B_y B_z \, dx \, dy| + |\int_S (B_x^2 + B_y^2) - B_z^2 \, dx \, dy|}{\int_S (B_x^2 + B_y^2 + B_z^2) \, dx \, dy}$$

3. The torque balance parameter $\epsilon_{\text{torque}} =$

$$\frac{|\int_S x((B_x^2 + B_y^2) - B_z^2) dx dy| + |\int_S y((B_x^2 + B_y^2) - B_z^2) \, dx dy| + |\int_S y B_x B_z - x B_y B_z dx dy|}{\int_S \sqrt{x^2 + y^2} \, (B_x^2 + B_y^2 + B_z^2) \, dx \, dy}$$

Magneto-Statics (MHS)

$$\mathbf{j} \times \mathbf{B} = \nabla P + \rho \nabla \Psi$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}, \ \nabla \cdot \mathbf{B} = 0$$

Several nonlinear force-free extrapolation codes have been generalized to include plasma forces:

- Optimization (Wiegelmann et. al. 2006, Zhu et al.2018,2019,2022)
- MHD-relaxation (Zhu et al. 2013)
- Grad-Rubin method (Gilchrist et al. 2016)

Mathematically simpler are linear MHS-models (Low 1991)

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} + a \exp(-\kappa z) \nabla B_z \times \mathbf{e_z}$$

Linearized MHS-equations

- Here we use a Cartesian system with (x,y) parallel and z perpendicular to the Sun‘s surface.
- Assumption: Currents flow in the x,y plane[perpendicular to gravity] + optional a linear current parallel to the field lines (Low 1991):

$$\underbrace{\nabla \times \mathbf{B} = \alpha_0 \mathbf{B}}_{\text{Linear force-free part}} + \underbrace{f(z) \nabla B_z \times \mathbf{e_z}}_{\text{this part contains currents perpendicular to z} \Rightarrow \text{nonmagnetic forces}}$$

Linear force-free part

this part contains currents perpendicular to z
=>nonmagnetic forces

Same decomposition is possible in spherical geometry (Bogdan&Low 86, Neukirch 95)

Linear MHS, Low 1991 solutions

$$\nabla \times \mathbf{B} = \alpha_0 \mathbf{B} + f(z) \nabla B_z \times \mathbf{e_z}$$

- The solar atmosphere becomes (almost) force-free above chromosphere [say thickness ~ 1/k], and the perpendicular part of the current should vanish in the corona:

$$f(z) = a \exp(-kz)$$

- With the measured Bz(x,y,z=0) in the photosphere as boundary condition, the equation above is solved with a Fast Fourier Transformation. **α_0** and **a** are free parameters.

How to constrain free parameters?

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} + a \exp(-\kappa z) \nabla B_z \times \mathbf{e_z}$$

$$\alpha = \frac{\sum \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \mathrm{sign}(B_z)}{\sum |B_z|}$$

Formula for Linear force-free fields (Hagino&Sakurai 2004)

$$\oint T \mathbf{ds}$$

Write Maxwell Stress Tensor (Forces) in components

This is only a first guess for a. We optimize a by comparing Eps_force from model with Eps_force from vector magnetogram.

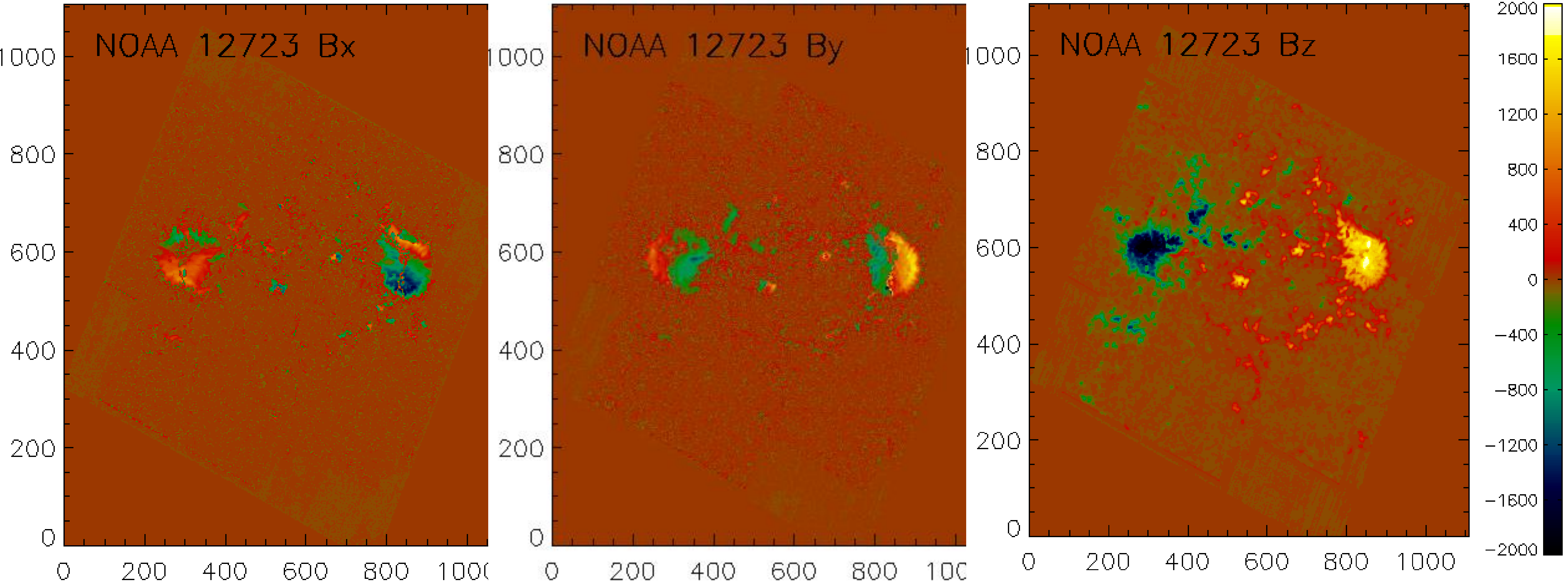
$$a = \frac{\left|\sum B_x B_z\right| + \left|\sum B_y B_z\right| + \left|\sum (B_x^2 + B_y^2) - B_z^2\right|}{\frac{1}{2}\sum (B_x^2 + B_y^2 + B_z^2)}$$

NOAA 12723

- nx=1108, ny=1108, nz=256, pix size: 85 km
- Flux imbalance: 0.008
- E_force: 0.410
- E_torque: 0.390
- Global Alpha: 2.272
- We compute 4 models:
  Potential Field (Pot), Linear Force-Free Field (LFFF, alpha=2.5)
  Linear Magneto-Hydro-Static Field
  (LMHS, alpha=2.5, a=1.0, kappa=0.04 => E_force_MHS=0.38)
- Nonlinear Force-Free Field (NLFFF, 1088 x 1088 x 256)
  The data are not ideal for NLFFF and the result is close to a potential field). Need better calibration?

Models in idl sav-files

- Bpot_full.sav Potential field, full box (1108x1108x256)
- Bff_full.sav Linear force-free field with alpha=2.5, full box (1108x1108x256)
- Bmhs_full.sav Linear magneto-hydro-static field with alpha=2.5, a=1.0, kappa=0.04, full box (1108x1108x256)
- BNLFFF_prepro.sav Nonlinear force-free field. Vectormagnetogram was preprocessed (including smoothing) and a 5-level multgrid was used, almost full box (1088x1088x256)



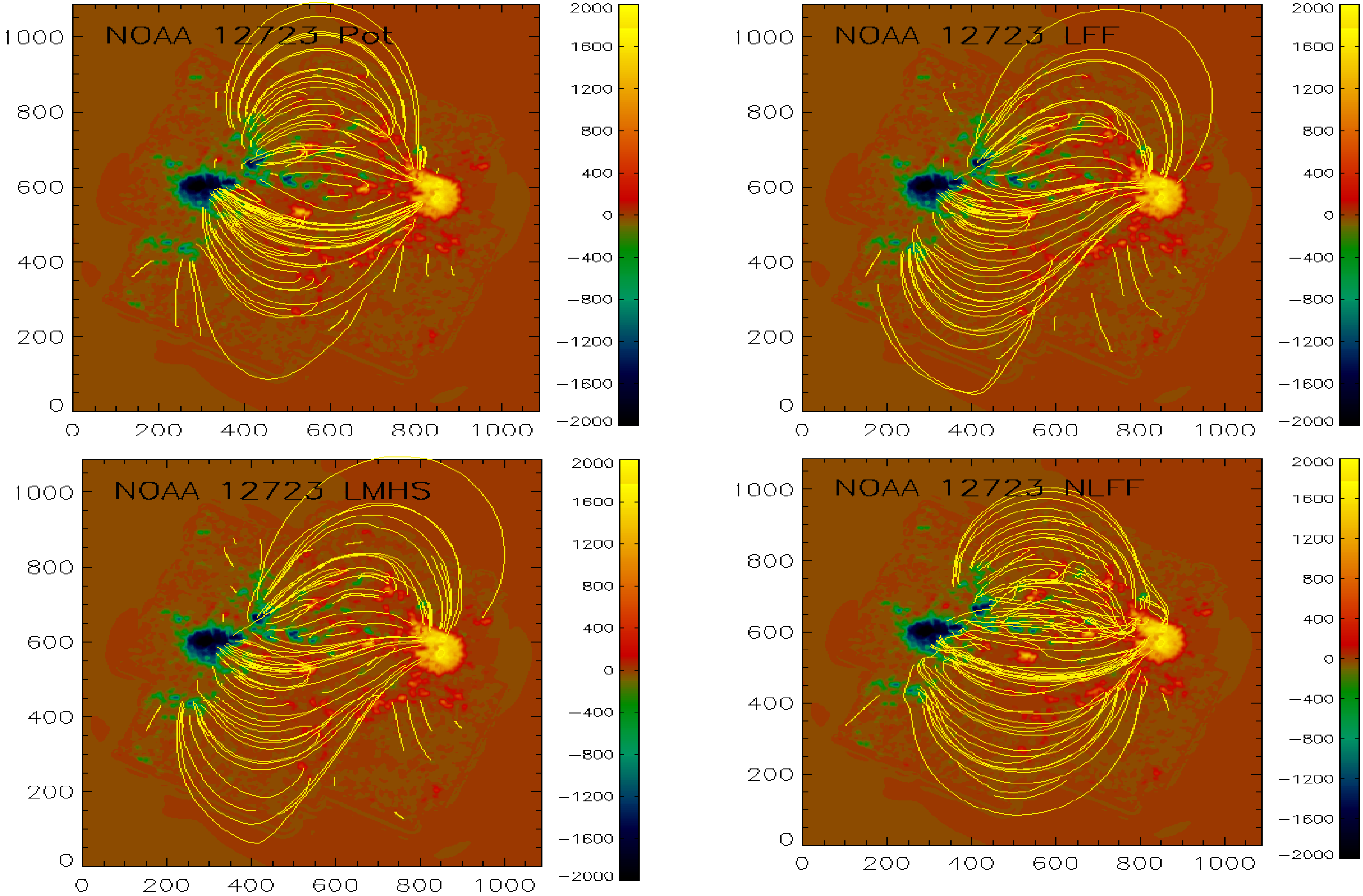
From João M. da Silva Santos:

The .fits data are at: https://next.issibern.ch/f/2096083

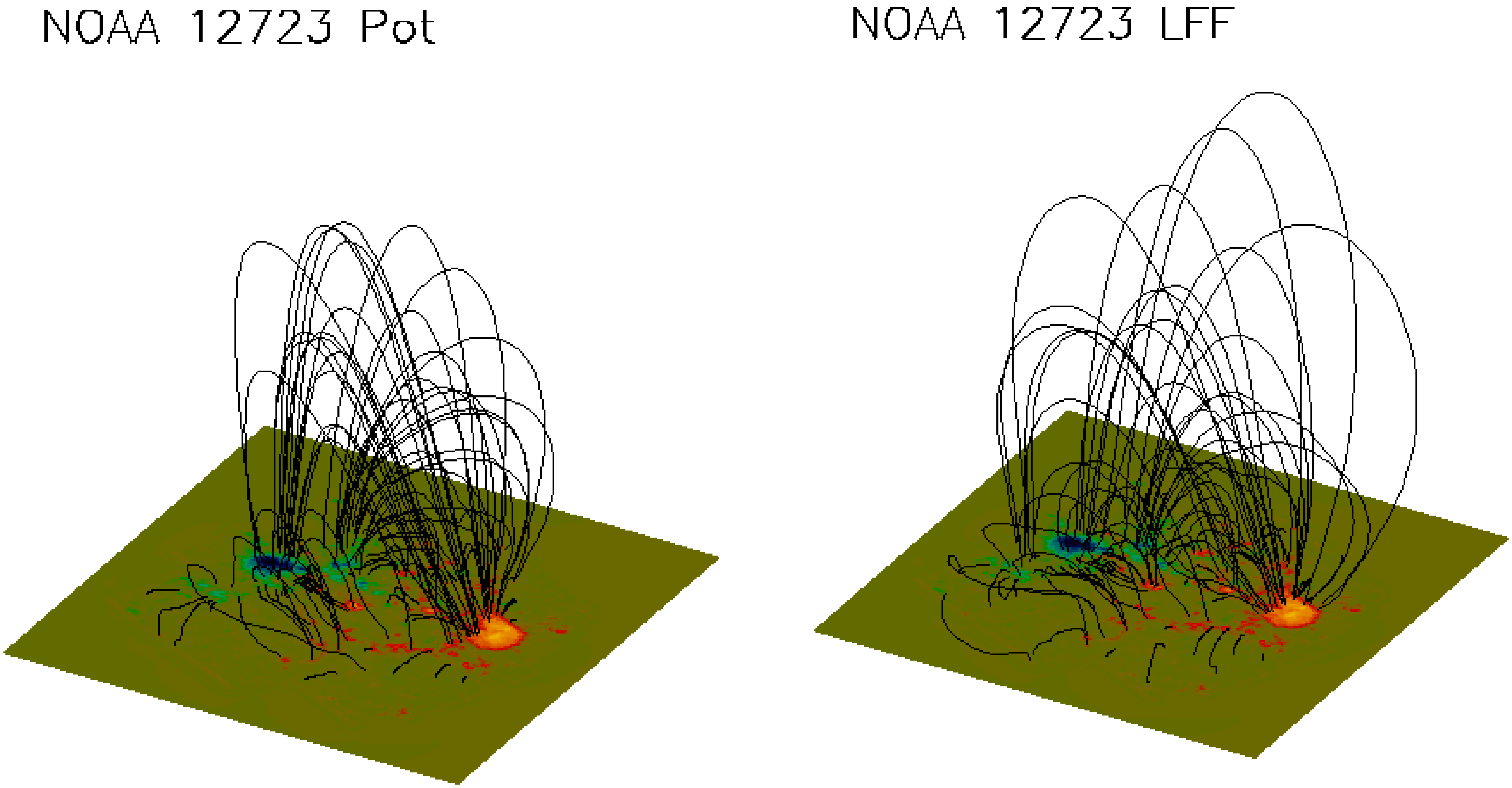
These results are preliminary, but we hope to have better-calibrated maps in the future.

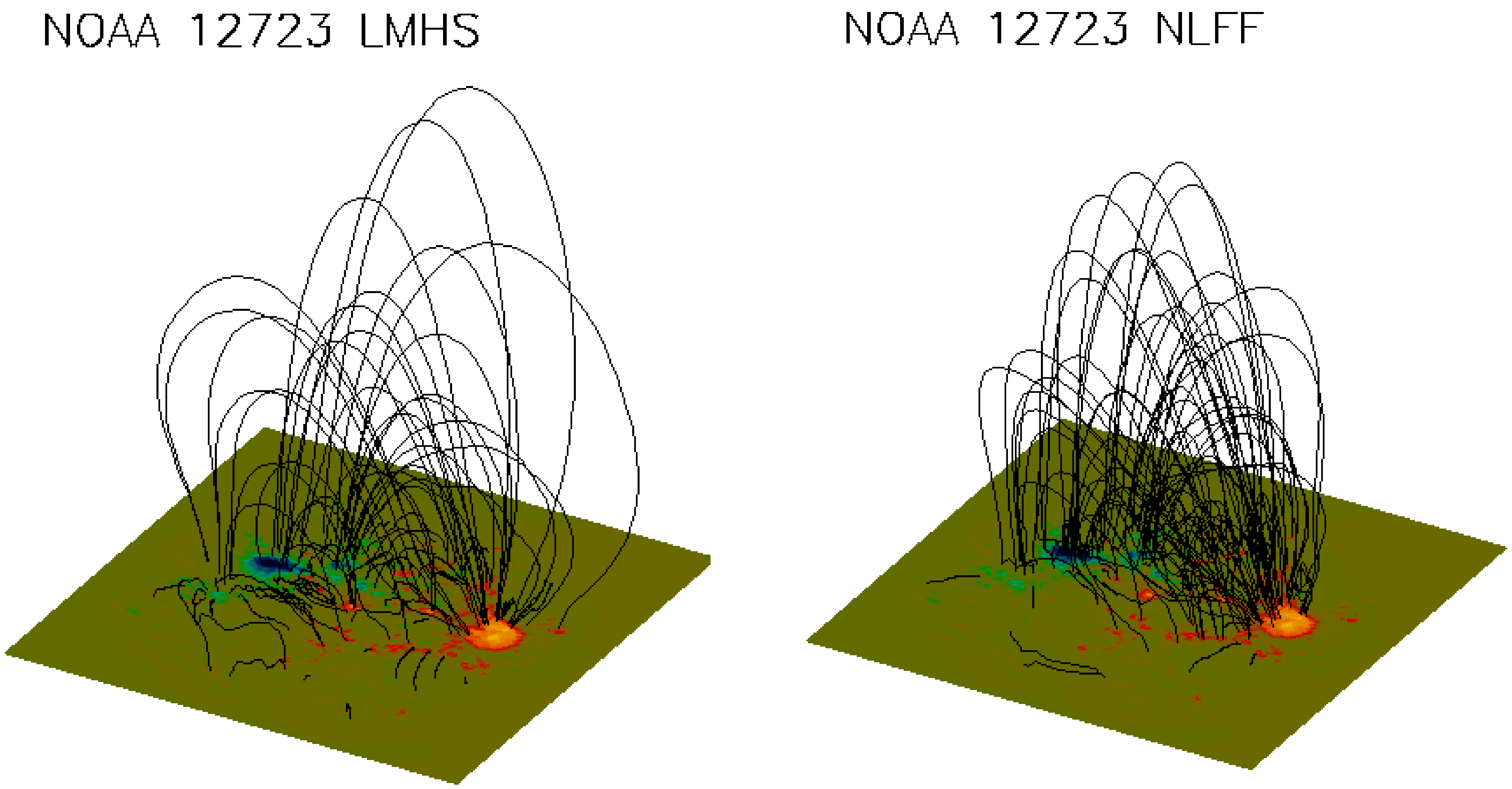
The pixel scale (0.118 arcsec) and pointing info can be read from the header in the BZ cube.

The data have not been projected to CEA coordinates.

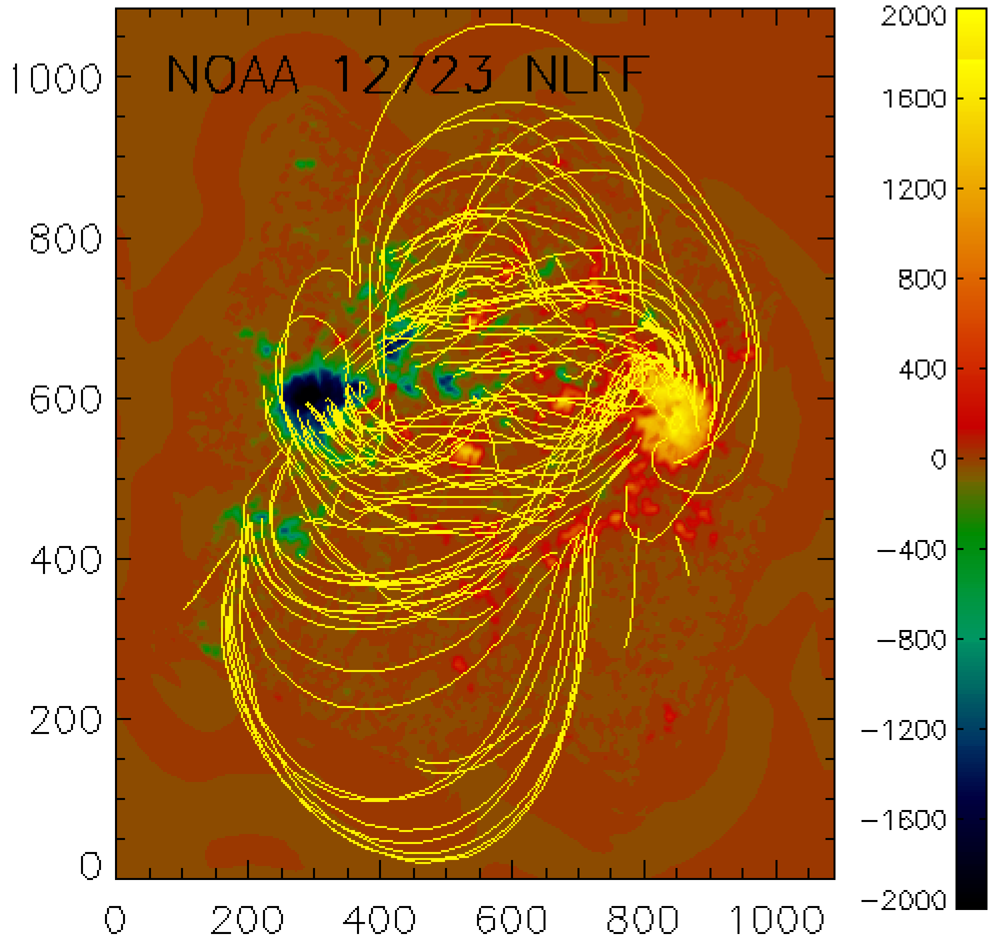


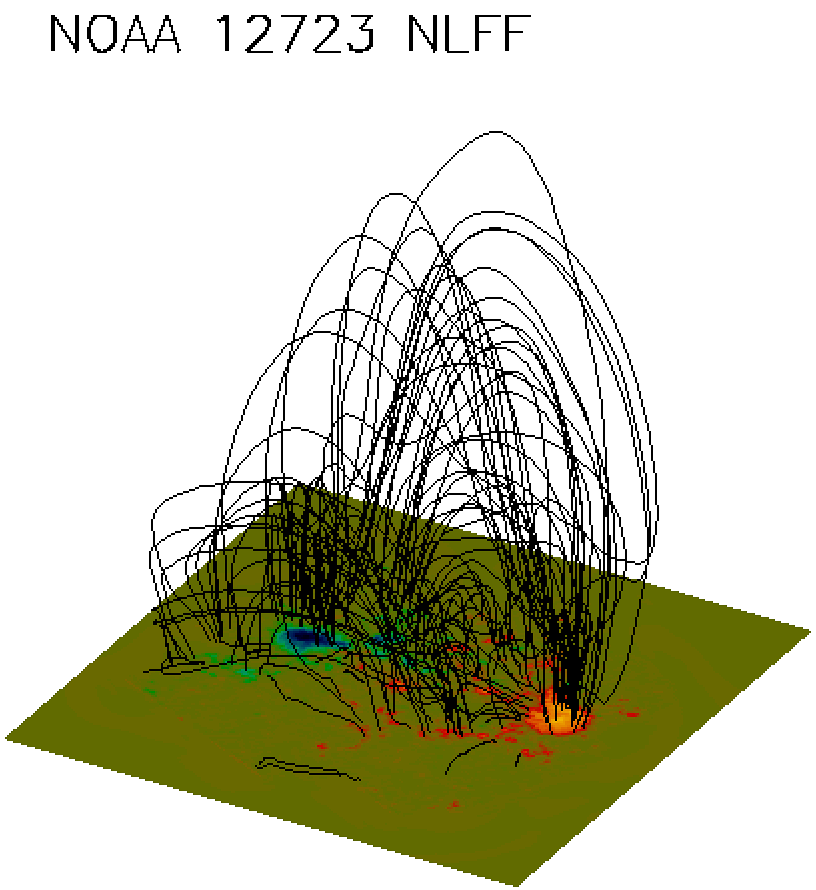
NOAA 12723 Pot
NOAA 12723 LFF
NOAA 12723 LMHS
NOAA 12723 NLFF
0
200
400
600
800
1000
2000
1600
1200
800
400
−400
−800
−1200
−1600
−2000


NOAA 12723 Pot
NOAA 12723 LFF


NOAA 12723 LMHS
NOAA 12723 NLFF

Some experiments with preprocessing





Energy very high: E_NLFF=1.58 E_pot